FORMULATION OF DESIGN TABLES AND CHARTS FOR DESIGN OF REINFORCED CONCRETE BEAMS

BY

MADU SOMTO JOHN

NAU/2017224051

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CERTIFICATION

This is to certify that the research project on "FORMULATION OF DESIGN TABLES AND CHARTS FOR DESIGN OF REINFORCED CONCRETE BEAMS" was carried out by Madu Somto John with registration number (NAU/2017224051) of the department of Civil Engineering in partial fulfillment of the requirement for the award of Bachelor's Degree in Civil Engineering, Nnamdi Azikiwe University Awka under the close supervision of Engr. Dr. Victor Okonkwo of Department of Civil Engineering, Nnamdi Azikiwe University, Awka. This work has never been submitted either in part or in full for any degree in any university.

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Madu Somto John   Date

(Student)

APPROVAL PAGE

This research thesis on "Formulation of design tables and charts for design of reinforced concrete beams" carried out by Madu Somto John with registration number (NAU/2017224051) has satisfied all the requirements of this university for the award of Bachelor's degree (B.Eng) in Civil Engineering, Nnamdi Azikiwe University, Awka.

________________________ ________________________

Engr. Dr. Victor Okonkwo Date

(Project Supervisor)

________________________ ________________________

Engr. Prof. A.C. Ezeagu. Date

(Head of Department)

________________________ ________________________

Engr. Prof. D.O. Onwuka. Date

(External Examiner)

DEDICATION

This Research project is dedicated to Almighty God, the seat of wisdom, to whom all things are possible.

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ABSTRACT

The need to reduce design rigors in the design of singly and doubly reinforced concrete beams formed the basis for the study. The study was undertaken to formulate design tables and charts for design of singly and doubly reinforced concrete beams with varying cross section. The developed design tables and charts were based on mathematical model derived in accordance with Euro code 2 design code of practice for reinforced concrete structures. The study also highlighted detailed process used in the design of reinforced concrete beams at both ultimate and serviceability limit state. Several design examples were solved using the developed design tables and charts. The accuracy of the developed design tables and charts was assessed through comparison with Euro code 2 design solutions. It was concluded that the developed design tables and charts was structurally reliable and efficient in the design of singly and doubly reinforced concrete beams as reasonable agreement exist between the output obtained in the design of singly and doubly reinforced concrete beams using the developed design tables and charts and Euro code 2 design code of practice.

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LIST OF SYMBOL& ABBREVIATION

SRCB – Singly Reinforced Concrete Beams

DRCB- Doubly Reinforced Concrete Beams

ULS – Ultimate Limit State

SLS- Serviceability Limit State

LS ---Limit State

DL –Dead Load

LL – Live load

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CHAPTER ONE

INTRODUCTION

1.1 Background of Study

Beams are horizontal structural elements designed to carry lateral loads (Oyenuga, 2011). When beams are inclined or slanted, they are referred to as raker beams (Oyenuga, 2011). Floor beams in a reinforced concrete structure are normally designed to resist loads from floor slabs, their own self weight, weight of finishes and other actions as may be applied. The design of reinforced concrete beams involves selection of proper beam size (cross section) and area of reinforcement to carry the applied loads without failing or deflecting excessively (Oyenuga, 2011). The analysis of beam cross section, determination of lever arm factor, percentage reinforcement, and design concrete shear stress involves the use of design tables and charts (BS 8110, 1985).

Reinforced concrete beams are divided into two namely; singly and doubly reinforced concrete beams. Singly reinforced concrete beams are beams that are reinforced only in the tension zone, in such beams, the maximum bending moment and tensile forces are borne by the reinforcement while the compressive forces are borne by the concrete (Oyenuga, 2011). In other hand, doubly reinforced concrete beams are beams with reinforcement at both the tension and compression zone, it is mainly provided when the cross section (beam depth) is limited. In such beams, the maximum bending moment, tensile and compressive forces are borne by the reinforcement. From economic point of view, design of doubly reinforced concrete beams is rarely considered except in special circumstances (where beam cross section is not limited).

Design of both single and doubly reinforced concrete beams involves the use of design tables and charts specified by relevant code of practice. The design of reinforced concrete beams with varying cross sections involves estimation of loads, analysis to determine the internal stresses (maximum bending moment and shear force), design of the beam section to determine the area of reinforcement and also serviceability limit checks of deflection, shear force, local and anchorage bond length. This is entirely a laborious and rigorous process especially when reinforced concrete beams with varying cross sections are to be designed. This is one of the problems faced by structural analysis and design engineers in the design of both singly and doubly reinforced concrete beams. It is however paramount to ensure efficiency and simplify the design process for design of both singly and doubly reinforced concrete beams. This can only be achieved through the formulation of simplified design tables and charts for determining the area of reinforcements for reinforced concrete beams at ultimate limit state of design.

Design charts cannot be used to obtain the complete detailed design of any member but they may be used as an aid when analyzing the cross section of a member at the ultimate limit state (BS 8110, 1985). In other to ensure efficiency in design of reinforced concrete beams (singly and doubly reinforced) and promote alternative design in reinforced concrete beams, this study will therefore develop simplified design tables and charts for specification of area of reinforcement at ultimate limit state.

1.2 Statement of Problem

Reinforced concrete beams are essential structural components of buildings, bridges, culverts and other civil engineering projects. Reinforced concrete beams are divided into singly and doubly reinforced concrete beams (Oyenuga, 2011). Analysis and design of both singly and doubly reinforced concrete beams involves the use of design tables and charts. Design charts cannot be used to obtain the complete detailed design of any member but they may be used as an aid when analyzing the cross section of a member at the ultimate limit state (BS 8110, 1985).

There is a growing concern shared by professionals in the construction industry over the application of inordinate rigor and time in the design of reinforced concrete beams with varying cross section as specified by relevant design code of practice. Design of different beams cross section in order to specify the area of reinforcement and carry out checks at serviceability limit state is entirely a laborious process requiring substantial amount of time and energy. Application of inordinate rigor and time in the design of reinforced concrete beams with different cross section (thickness, span and width) has a potential of affecting design output thereby resulting to design flaws.

In other to ensure efficiency, accuracy and simplicity in design of reinforced concrete beams, this study will therefore formulate a simplified tables and charts for specification of percentage reinforcement at ultimate limit state of design.

1.3 Aim: Formulation of design tables and charts for the design of reinforced concrete beams

1.4 Objective: The objective of this study includes the followings:

i. Select and review design code of practice relevant to the design of reinforced concrete beams
ii. Review reinforced concrete design textbooks for the method of design of reinforced concrete beams
iii. Formulate the design steps for the design of reinforced concrete beams
iv. Develop graphs and tables for the implementation of the design steps
v. Demonstrate the use of the new graphs to assess their usefulness

1.5 Scope of Study

The study is purely analytical and is centered at formulation of design tables and charts for design of reinforced concrete beams with varying cross sections. Design tables and charts will be developed in accordance with the procedure stated in relevant code of practice (Euro code 2: design of concrete structures). Mathematical model which will aid in the formulation of design tables and charts for both singly and doubly reinforced concrete beams will be developed. Design tables and charts for singly reinforced concrete beams, doubly reinforced concrete beams and both singly and doubly reinforced concrete beams (combined) will be developed. Two singly and doubly reinforced beams with varying cross section (thickness, span and width) usually of standard value will be designed using the developed design charts and tables and Euro code 2 design code of practice. Detailed step by step procedures of determining the aforementioned tables and charts will also be presented with explicit explanation on their application during design of reinforced concrete beams with standard cross section. Accuracy of developed design tables and charts will be assessed and deviation from standard will be analyzed.

1.6 Significance of Study

The findings obtained from the study on formulation of simplified design tables and charts for design of reinforced concrete beams with varying cross section will however be significant in the following ways:

1. Ensure simplicity and efficiency in design of reinforced singly and doubly reinforced concrete beams with specific cross section.
2. Ensure regular update of material factor of safety for design of singly and doubly reinforced concrete beams.
3. Serve as a reference or body of knowledge for undergraduate and design professionals in the construction industry.
4. Ensure optimization in the design of both singly and doubly reinforced concrete beams.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Overview

Beams are horizontal structural elements designed to carry lateral loads (Oyenuga, 2011). When beams are inclined or slanted, they are referred to as raker beams (Oyenuga, 2011). Floor beams in a reinforced concrete structure are normally designed to resist loads from floor slabs, their own self weight, weight of finishes and other actions as may be applied. According to (Oyenuga, 2011), reinforced concrete members in building can be categorized into horizontal members which include slabs, beams and staircases with the adjoining landing and vertical members which include columns and walls. Loads from slab, walls are transmitted through the beams to the column and finally to the foundation below. Apart from being horizontal in plan, a beam may be slant (ranked beam) or circular (arcate) in plan. A beam like any other member is designed to resist the ultimate bending moment, shear force and torsional moments if any. Also, serviceability requirements must be adequately considered to ensure that the member will behave satisfactorily under the working loads. However, much emphasis is generally placed on the ultimate limit state requirements of beams. The serviceability limit state is being taken care off through the span/effective depth ratio.

Reinforced concrete beams are divided into two namely; singly and doubly reinforced concrete beams. Singly reinforced concrete beams are beams that are reinforced only in the tension zone, in such beams, the maximum bending moment and tensile forces are borne by the reinforcement while the compressive forces are borne by the concrete (Oyenuga, 2011). In other hand, doubly reinforced concrete beams are beams with reinforcement at both the tension and compression zone, it is mainly provided when the cross section (beam depth) is limited. In such beams, the maximum bending moment, tensile and compressive forces are borne by the reinforcement. From economic point of view, design of doubly reinforced concrete beams is rarely considered except in special circumstances (where beam cross section is not limited).

This section will review relevant literatures on reinforced concrete beams and principles for formulation of design tables and charts for reinforced concrete beams.

2.2 Historical Facts about Reinforced Concrete

Many researchers believe that the first use of a truly cementitious binding agent (as opposed to the ordinary lime commonly used in ancient mortars) occurred in southern Italy around second century BC. Volcanic ash (called pozzuolana, found near Pozzouli, by the Bay of Naples) was a key ingredient in the Roman cement used during the days of the Roman Empire. Roman concrete bears little resemblance to modern Portland cement concrete. It was never put into a mould or formwork in a plastic state and made to harden, as is being done today. Instead, Roman concrete was constructed in layers by packing mortar by hand in and around stones of various sizes. The Pantheon, constructed in AD 126, is one of the structural marvels of all times (Shaeffer 1992).

During the middle Ages, the use of concrete declined, although isolated instances of its use have been documented and some examples have survived. Concrete was more extensively used again during the Renaissance (14th–17th centuries) in structures like bridge piers. Pozzolanic materials were added to the lime, as done by the Romans, to increase its hydraulic properties (Reed, et al. 2008). In the eighteenth century, with the advent of new technical innovations, a greater interest was shown in concrete. In 1756, John Smeaton, a British Engineer, rediscovered hydraulic cement through repeated testing of mortar in both fresh and salt water. Smeaton's work was followed by Joseph Aspdin, a bricklayer and mason in Leeds, England, who, in 1824, patented the first 'Portland' cement, so named since it resembled the stone quarried on the Isle of Portland off the British coast (Reed, et al. 2008). Aspdin was the first to use high temperatures to heat alumina and silica materials, so that cement was formed. It is interesting to note that cement is still made in this way. I.K. Brunel was the first to use Portland cement in an engineering application in 1828; it was used to fill a breach in the Thames Tunnel. During 1959–67, Portland cement was used in the construction of the London sewer system.

The small rowboats built by Jean-Louis Lambot in the early 1850s are cited as the first successful use of reinforcements in concrete. During 1850–1880, a French builder, Francois Coignet, built several large houses of concrete in England and France (Reed, et al. 2008). Joseph Monier of France, who is considered to be the first builder of RC, built RC reservoirs in 1872. In 1861, Monier published a small book, Das System Monier, in which he presented the applications of RC. During 1871–75, William E. Ward built the first landmark building inRC in Port Chester, NY, USA. In 1892, François Hennebique of France patented a system of steel-reinforced beams, slabs, and columns, which were used in the construction of various structures, built in England between 1897 and 1919. In Hennebique's system, steel reinforcement was placed correctly in the tension zone of the concrete; this was backed by a theoretical understanding of the tensile and compressive forces, which was developed by Cottançin in France in 1892 (Reed, et al. 2008).

2.3 Reinforced Concrete Design Criteria and Safety Consideration

A limit-state design concept is used in British and European Codes of Practice. Ultimate (ULS) and serviceability (SLS) limit states need to be considered as well as durability and, in the case of buildings, fire-resistance. Partial safety factors are incorporated into loads (including imposed deformations) and material strengths to ensure that the probability of failure (not satisfying a design requirement) is acceptably low (Reynolds and Steedman, 2008).

In BS 8110 at the ULS, a structure should be stable under all combinations of dead, imposed and wind load. It should also be robust enough to withstand the effects of accidental loads, due to an unforeseen event such as a collision or explosion, without disproportionate collapse. At the SLS, the effects in normal use of deflection, cracking and vibration should not cause the structure to deteriorate or become unserviceable. A deflection limit of span/250 applies for the total sag of a beam or slab relative to the level of the supports. A further limit, the lesser of span/500 or 20 mm, applies for the deflection that occurs after the application of finishes, cladding and partitions so as to avoid damage to these elements. A limit of 0.3 mm generally applies for the width of a crack at any point on the concrete surface.

In BS 5400, an additional partial safety factor is introduced. This is applied to the load effects and takes account of the method of structural analysis that is used. Also there are more load types and combinations to be considered. At the SLS, there are no specified deflection limits but the cracking limits are more critical. Crack width limits of 0.25, 0.15 or 0.1 mm apply according to surface exposure conditions. Compressive stress limits are also included but in many cases these do not need to be checked. Fatigue considerations require limitations on the reinforcement stress range for unwelded bars and more fundamental analysis if welding is involved. Footbridges are to be analyzed to ensure that either the fundamental natural frequency of vibration or the maximum vertical acceleration meets specified requirements.

In BS 8007, water-resistance is a primary design concern. Any cracks that pass through the full thickness of a section are likely to allow some seepage initially, resulting in surface staining and damp patches. Satisfactory performance depends upon autogenous healing of such cracks taking place within a few weeks of first filling in the case of a containment vessel. A crack width limit of 0.2 mm normally applies to all cracks, irrespective of whether or not they pass completely through the section. Where the appearance of a structure is considered to be aesthetically critical, a limit of 0.1 mm is recommended.

2.4 Reinforced Concrete Design Process

There are two principal stages in the calculations required to design a reinforced concrete structure. In the first stage, calculations are made to determine the effect on the structure of loads and imposed deformations in terms of applied moments and forces. In the second stage, calculations are made to determine the capacity of the structure to withstand such effects in terms of resistance moments and forces (Reynolds and Steedman, 2008).

Factors of safety are introduced in order to allow for the uncertainties associated with the assumptions made and the values used at each stage. For many years, un-factored loads were used in the first stage and total factors of safety were incorporated in the material stresses used in the second stage. The stresses were intended to ensure both adequate safety and satisfactory performance in service. This simple approach was eventually replaced by a more refined method, in which specific design criteria are set and partial factors of safety are incorporated at each stage of the design process. In modern Codes of Practice, a limit-state design concept isused. Ultimate (ULS) and serviceability (SLS) limit-states are considered, as well as durability and, in the case of buildings, fire-resistance. Partial safety factors are incorporated in both loads and material strengths, to ensure that the probability of failure (i.e. not satisfying a design requirement) is acceptably low.

Members are first designed to satisfy the most critical limit state, and then checked to ensure that the other limit-states are not reached. For most members, the critical condition to be considered is the ultimate limit state (ULS), on which the required resistances of the member in bending, shear and torsion are based. The requirements of the various serviceability limit state (SLS), such as deflection and cracking, are considered later. However, since the selection of an adequate span to effective depth ratio to prevent excessive deflection, and the choice of a suitable bar spacing to avoid excessive cracking, can also be affected by the reinforcement stress, the design process is generally interactive. Nevertheless, it is normal to start with the requirements of the ultimate limit state (ULS).

2.5 Reinforced Concrete Loadings

The loads (actions) acting on a structure generally consist of a combination of dead (permanent) and live (variable) loads. In limit-state design, a design load (action) is calculated by multiplying the characteristic (or representative) value by an appropriate partial factor of safety. The characteristic value is generally a value specified in a relevant standard or code. In particular circumstances, it may be a value given by a client or determined by a designer in consultation with the client. In BS 8110 characteristic dead, imposed and wind loads are taken as those defined in and calculated in accordance with BS 6399: Parts 1, 2 and 3. In BS 5400 characteristic dead and live loads are given in Part 2, but these have been superseded in practice by the loads in the appropriate Highways Agency standards.

When Euro code 2 (EC 2: Part 1.1) was first introduced as an ENV document, characteristic loads were taken as the values given in BS 6399 but with the specified wind load reduced by 10%. This was intended to compensate for the partial safety factor applied to wind at the ultimate limit state (ULS) being higher in the Eurocodes than in BS 8110.

2.5.1 Dead Loads (Permanent Action)

Dead loads include the weights of the structure itself and all permanent fixtures, finishes, surfacing and so on. When permanent partitions are indicated, they should be included as dead loads acting at the appropriate locations. Where any doubt exists as to the permanency of the loads, they should be treated as imposed loads. Dead loads can be calculated from the unit weights given in EC 1: Part 1.1, or from actual known weights of the materials used.

2.5.2 Live Loads (Variable Action)

Live loads comprise any transient external loads imposed on the structure in normal use due to gravitational, dynamic and environmental effects. They include loads due to occupancy (people, furniture, moveable equipment), traffic (road, rail, pedestrian), retained material (earth, liquids, granular), snow, wind, temperature, ground and water movement, wave action and so on. Careful assessment of actual and probable loads is a very important factor in producing economical and efficient structures. Some imposed loads, like those due to contained liquids, can be determined precisely. Other loads, such as those on floors and bridges are very variable. Snow and wind loads are highly dependent on location.

2.6 Review of Reinforced Concrete Design Methods

2.6.1 Elastic Methods

The so-called exact theory of the elastic bending of plates spanning in two directions derives from work by Lagrange, who produced the governing differential equation for plate bending in 1811, and Navier, who in 1820 described the use of a double trigonometric series to analyze freely supported rectangular plates. Pigeaud and others later developed the analysis of panels freely supported along all four edges. Many standard elastic solutions have been produced but almost all of these are restricted to square, rectangular and circular. Exact analysis of a beams having an arbitrary shape and support conditions with a general arrangement of loading would be extremely complex. To deal with such problems, numerical techniques such as finite differences and finite elements have been devised.

2.6.2 Collapse Methods

Unlike in frame design, where the converse is generally true, it is normally easier to analyze beams by collapse methods than by elastic methods. The most-widely known methods of plastic analysis of beams are the yield-line method developed by K W Johansen, and the so-called strip method devised by Arne Hillerborg (Reynolds and Steedman, 2008). It is generally impossible to calculate the precise ultimate resistance of a beam by collapse theory, since such elements are highly indeterminate. Instead, two separate solutions can be found, one being upper bound and the other lower bound. With solutions of the first type, a collapse mechanism is first postulated. Then, if the beam is deformed, the energy absorbed in inducing ultimate moments along the yield lines is equal to the work done on the beam by the applied load in producing this deformation. Thus, the load determined is the maximum that the beam will support before failure occurs. However, since such methods do not investigate conditions between the postulated yield lines to ensure that the moments in these areas do not exceed the ultimate resistance of the slab, there is no guarantee that the minimum possible collapse load has been found. This is an inevitable shortcoming of upper-bound solutions such as those given by Johansen’s theory.

Conversely, lower-bound solutions will generally result in the determination of collapse loads that are less than the maximum that the beam can actually carry. The procedure here is to choose a distribution of ultimate moments that ensures that equilibrium is satisfied throughout, and that nowhere is the resistance of the beam exceeded (Reynolds and Steedman, 2008).. Most of the literature dealing with the methods of Johansen and Hillerborg assumes that any continuous supports at the beam edges are rigid and unyielding.

2.6.3 Yield Line Analysis Method

Johansen's method requires the designer to first postulate an appropriate collapse mechanism for the slab being considered according to the rules given in section 13.4.2 in Reinforced Concrete Design Manual by (Reynolds and Steedman, 2008). Variable dimensions may then be adjusted to obtain the maximum ultimate resistance for a given load (i.e. the maximum ratio of M/F). This maximum value can be found in various ways, using actual numerical values and employing a trial and adjustment process. The work equation may be expressed algebraically and, by substituting various values for the maximum ratio of M/F may be read from a graph relating ó to M/F. Another method is to use calculus to differentiate the equation and then, by setting this equal to zero, determine the critical value of ó.

Yield-line theory is too complex to deal with adequately (Reynolds and Steadman, 2008). Indeed, several textbooks are completely or almost completely devoted to the subject In section 13.4 and Tables 2.49 and 2.50, of Reinforced Concrete Design Handbook by (Reynolds and Steadman, 2008), notes and examples are given on the rules for choosing yield-line patterns for analysis, on theoretical and empirical methods of analysis, on simplifications that can be made by using so-called affinity theorems, and on the effects of corner levers.

2.6.4 Strip Method

Hillerborg devised his strip method in order to obtain a lower-bound solution for the collapse load, while achieving a good economical arrangement of reinforcement. As long as the reinforcement provided is sufficient to cater for the calculated moments, the strip method enables such a lower-bound solution to be obtained. (Hillerborg and others sometimes refer to the strip method as the equilibrium theory; this should not, however, be confused with the equilibrium method of yield-line analysis.) In Hillerborg's original theory (now known as the simple strip method), it is assumed that, at failure, no load is resisted by torsion and thus, all load is carried by flexure in either of two principal directions. The theory results in simple solutions giving full information regarding the moments over the whole beam to resist a unique collapse load, the reinforcement being placed economically in bands. Brief notes on the use of simple strip theory to design rectangular beams supporting uniform loads are given in section 13.5 and Table 2.51 in Reinforced Concrete Design Handbook by (Reynolds and Steadman, 2008).

However, the simple strip theory is unable to deal with concentrated loads and/or supports and leads to difficulties with free edges. To overcome such problems, Hillerborg later developed his advanced strip method, which involves the use of complex moment fields. Although this development extends the scope of the simple strip method, it somewhat spoils the simplicity and directness of the original concept. A further disadvantage of both Hillerborg's and Johansen's methods is that, being based on conditions at failure only, they permit unwary designers to adopt load distributions that may differ widely from those that would occur under service loads, with the risk of unforeseen cracking. A development that eliminates this problem, as well as overcoming the limitations arising from simple strip theory, is the so-called strip-deflection (Reynolds and Steedman, 2008). With this method the distribution of load in either principal direction is not selected arbitrarily by the designer (as in the Hillerborg method or, by choosing the ratio of reinforcement provided in each direction, as in the yield-line method) but is calculated so as to ensure compatibility of deflection in mutually orthogonal strips. The method results in sets of simultaneous equations (usually eight), the solution of which requires computer assistance (Reynolds and Steedman, 2008).

2.6.5 Limit State Method

In limit state method of reinforced concrete design, the working loads are multiplied by partial factor of safety and the ultimate material strength are divided by further partial factor of safety (Oyenuga, 2011). The limit state design philosophy, which was formulated for reinforced concrete design in Russia during the 1930s, achieves the objectives set out in Section 2.3 by considering two 'types' of limit state under which a structure may become unfit for its intended purpose. They include:

2.6.5.1 Serviceability Limit State: This state ensures satisfactory behaviour under service loads. The principal criteria relating to serviceability are the prevention of deflection, vibration or cracking. Under consideration of strength, the bridge members need to be strong enough to withstand the live and dead loads identified above with an adequate margin of safety to allow for uncertainties in loading, material properties and quality of construction and maintenance. Deflection requirement suggest that he pedestrian bridge should not deflect to an extent that might cause concern or discomfort to users or cause fixed members to become out of plane. Maximum limits for beam and truss pedestrian bridges range from span/180 (5.5mm per m of span) to span/360 (2.75mm per of span). A middle value of span/250 (4mm per m of span) is used in this manual. The limit is the maximum deflection at the Centre of the pedestrian bridge when loaded by the above live loads.

2.6.5.2 Ultimate Limit State: This design state ensures that the probability of failure is acceptably low and the structure, or some part of it, is safe for its intended purpose (Oyenuga, 2011).

2.7 Reinforced Concrete Beams

Beams are horizontal structural elements designed to carry lateral loads (Oyenuga, 2011). When beams are inclined or slanted, they are referred to as raker beams (Oyenuga, 2011). Floor beams in a reinforced concrete structure are normally designed to resist loads from floor slabs, their own self weight, weight of finishes and other actions as may be applied. According to (Oyenuga, 2011), reinforced concrete members in building can be categorized into horizontal members which include slabs, beams and staircases with the adjoining landing and vertical members which include columns and walls. Loads from slab, walls are transmitted through the beams to the column and finally to the foundation below. Apart from being horizontal in plan, a beam may be slant (ranked beam) or circular (arcate) in plan. A beam like any other member is designed to resist the ultimate bending moment, shear force and torsional moments if any. Also, serviceability requirements must be adequately considered to ensure that the member will behave satisfactorily under the working loads. However, much emphasis is generally placed on the ultimate limit state requirements of beams. The serviceability limit state is being taken care off through the span/effective depth ratio.

2.7.1 Review of Reinforced Concrete Beam Types

2.7.1 Continuous Beams

A beam with more than simple supports is a continuous beam (Oyenuga, 2011). Continuous and other beams with only transverse loads, with more than two reaction components are called statically indeterminate since there are not enough equations of equilibrium to determine the reactions.

Historically, various methods of structural analysis have been developed for determining the bending moments and shearing forces on beams continuous over two or more spans (Reynolds and Steedman, 2008). Most ofthese have been stiffness methods, which are generally better suited than flexibility methods to hand computation. Some of these approaches, such as the theorem of three-moments and the methods of fixed points and characteristic points, were included in the previous edition of this Handbook. If beams having two, three or four spans are of uniform cross section, and supportloads that are symmetrical on each individual span, formulae and coefficients can be derived that enable the support moments to be determined by direct calculation. Such a method is given in Table 2.37 in Reinforced Concrete Design Manual by (Reynolds and Steedman, 2008). More generally, in order to avoid the need to solve large sets of simultaneous equations, methods involving successive approximations have been devised. Despite the general use of computers, hand methods can still be very useful in dealing with routine problems. The ability to use hand methods also develops in the engineer an appreciation of analysis that is invaluable in applying output from the computer.

When bending moments are calculated with the spans taken as the distances between the centers of supports, the critical negative moment in monolithic forms of construction can be considered as that occurring at the edge of the support (Reynolds and Steedman, 2008). When the supports are of considerable width, the span can be taken as the clear distance between the supports plus the effective depth of the beam, or an additional span can be introduced that is equal to the width of the support minus the effective depth of the beam. The load on this additional span should be taken as the support reaction spread uniformly over the width of the support. If a beam is constructed monolithically with a very wide and massive support, the effect of continuity with the span or spans beyond the support may be negligible, in which case the beam should be treated as fixed at the support.

The second moment of area of a reinforced concrete beam of uniform depth may still vary throughout its length, due to variations in the amount of reinforcement and also because, when acting with an adjoining slab, a down-stand beam may be considered as a flanged section at mid-span but a simple rectangular section at the supports. It is common practice, however, to neglect these variations for beams of uniform depth, and use the value of I for the plain rectangular section. It is often assumed that a continuous beam is freely supported at the ends, even when beam and support are constructed monolithically. Some provision should still be made for the effects of end restraint.

2.7.2 Doubly Reinforced Concrete Beams

Doubly reinforced concrete beams are beams with reinforcement at both the tension and compression zone, it is mainly provided when the cross section (beam depth) is limited. In such beams, the maximum bending moment, tensile and compressive forces are borne by the reinforcement. From economic point of view, design of doubly reinforced concrete beams is rarely considered except in special circumstances (where beam cross section is limited).

A section needing both tension and compression reinforcement, and subjected to a moment M, can be designed by first selecting a suitable value for x, such as the limiting value for using the maximum design stress in the tension reinforcement or satisfying the condition necessary for moment redistribution (Reynolds and Steedman, 2008). The required force to be provided by the compression reinforcement can be derived by taking moments, for the compressive forces in the concrete and the reinforcement, about the line of action of the tensile reinforcement. The force to be provided by the tension reinforcement is equal to the sum of the compressive forces. The reinforcement areas can now be determined, taking due account of the strains appropriate to the value of x selected.

2.7.3 Singly Reinforced Concrete Beams

Singly reinforced concrete beams are beams that are reinforced only in the tension zone, in such beams, the maximum bending moment and tensile forces are borne by the reinforcement while the compressive forces are borne by the concrete (Oyenuga, 2011). For a section that is reinforced in tension only, and subjected to a moment M, a quadratic equation in x can be obtained by taking moments, for the compressive force in the concrete, about the line of action of the tension reinforcement. The resulting value of x can be used to determine the strain diagram, from which the strain in the reinforcement, and hence the stress, can be calculated. The required area of reinforcement can then be determined from the tensile force, whose magnitude is equal to the compressive force in the concrete. If the calculated value of x exceeds the limit required for any redistribution of moment, then a doubly reinforced section will be necessary.

In designs to BS 8110 and BS 5400, the lever arm between the tensile and compressive forces is to be taken not greater than 0.95d. Furthermore, it is a requirement in BS 5400 that, if x exceeds the limiting value for using the maximum design stress, then the resistance moment should be at least 1.15M. Analyses are included in section 24.2.1 for both BS 8110 and BS 5400 and in section 32.2.1 for EC 2. Design charts based on the parabolic-rectangular stress-block for concrete, with fy (500 N/mm2), are given in Tables 3.13, 3.23 and 4.7 for BS 8110, BS 5400 and EC 2 respectively. Design tables based on the rectangular stress-blocks for concrete are given in Tables 3.14, 3.24 and 4.8 for BS 8110, BS 5400 and EC 2 respectively.

2.7.4 Cantilever Beams

A cantilever beam is one in which one end is built intoa wall or other support so that the built-in end cannot move transversely or rotate. Thebuilt-in end is said to be fixed if no rotation occurs and restrained if a limited amount ofrotation occurs. Cantilever beams and simple beams have two reactions (two forces or one force and a couple) and these reactions can be obtained from a free-body diagram of the beam by applying the equations of equilibrium. Such beams are said to be statically determinate since the reactions can be obtained from the equations of equilibrium.

2.8 Serviceability Checks for Reinforced Concrete Beams

2.8.1 Cracking

Cracks in members under service loading should not impair the appearance, durability or water-tightness of a structure. In BS 8110, for buildings, the design crack width is generally limited to 0.3 mm. In BS 5400, for bridges, the limit varies between 0.25 mm and 0.10 mm depending on the exposure conditions. In BS 8007, for structures to retain liquids, a limit of 0.2 mm usually applies. Under liquid pressure, continuous cracks that extend through the full thickness of a slab or wall are likely to result in some initial seepage, but such cracks are expected to self-heal within a few weeks. If the appearance of a liquid-retaining structure is considered aesthetically critical, a crack width limit of 0.1 mm applies.

In EC 2, for most buildings, the design crack width is generally limited to 0.3 mm, but for internal dry surfaces, a limit of 0.4 mm is considered sufficient. For liquid-retaining structures, a classification system according to the degree of protection required against leakage is introduced. Where a small amount of leakage is acceptable, for cracks that pass through the full thickness of the section, the crack width limit varies according to the hydraulic gradient (i.e. head of liquid divided by thickness of section). In order to control cracking in the regions where tension is expected, it is necessary to ensure that the tensile capacity of the reinforcement at yielding is not less than the tensile force in the concrete just before cracking. Thus a minimum amount of reinforcement is required, according to the strength of the reinforcing steel and the tensile strength of the concrete at the time when cracks may first be expected to occur. Cracks due to restrained early thermal effects in continuous walls and some slabs may occur within a few days of the concrete being placed. In other members, it may be several weeks before the applied load reaches a level at which cracking occurs.

Crack widths are influenced by several factors including the cover, bar size, bar spacing and stress in the reinforcement. The stress may need to be reduced in order to meet the crack widthlimit. Design formulae are given in Codes of Practice in which strain, calculated on the basis of no tension in the concrete, is reduced by a value that decreases with increasing amounts of tension reinforcement. For cracks that are caused by applied loading, the same formulae are used in BS 8110, BS 5400 and BS 8007. For cracks that are caused by restraint to temperature effects and shrinkage, fundamentally different formulae are included in BS 8007. Here, it is assumed that bond slip occurs at each crack, and the crack width increases in direct proportion to the contraction of the concrete.

2.8.2 Deflection

The deflections of members under service loading should not impair the appearance or function of a structure. An accurate prediction of deflections at different stages of construction may also be necessary in bridges, for example. For buildings, the final deflection of members below the support level, after allowance for any pre-camber, is limited to span/250. In order to minimize any damage to non-structural elements such as finishes, cladding or partitions that part of the deflection that occurs after the construction stage is also limited to span/500. In BS 8110, this limit is taken as 20 mm for spans ≥10 m.

The behaviour of a reinforced concrete beam under service loading can be divided into two basic phases: before and after cracking. During the un-cracked phase, the member behaves elastically as a homogeneous material. This phase is ended by the load at which the first flexural crack forms. The cracks result in a gradual reduction in stiffness with increasing load during the cracked phase. The concrete between the cracks continues to provide some tensile resistance though less, on average, than the tensile strength of the concrete (Reynolds and Steedman, 2008). Thus, the member is stiffer than the value calculated on the assumption that the concrete carries no tension. This additional stiffness, known as 'tension stiffening', is highly significant in lightly reinforced members such as slabs, but has only a relatively minor effect on the deflection of heavily reinforced members.

2.8.3 Lap Length

Forces can be transferred between reinforcement by lapping, welding or joining bars with mechanical devices (couplers). Connections should be placed, whenever possible, away from positions of high stress, and should preferably be staggered. In Codes of Practice, the necessary lap length is obtained by multiplying the required anchorage length by a coefficient.

In BS 8110, for bars in compression, the coefficient is 1.25. For bars in tension, the coefficient is 1.0, 1.4 or 2.0 according to the cover, the gap between adjacent laps in the same layer and the location of the bar in the section. In slabs, where the cover is not less than twice the bar size, and the gap between adjacent laps is not less than six times the bar size or 75 mm, a factor of 1.0 applies. Larger factors are frequently necessary in columns, typically 1.4; and beams, typically 1.4 for bottom bars and 2.0 for top bars. The sum of all the reinforcement sizes in a particular layer should not exceed 40% of the width of the section at that level. When the size of both bars at a lap exceeds 20 mm, and the cover is less than 1.5 times the size of the smaller bar, links at a maximum spacing of 200 mm are required throughout the lap length.

In EC 2, for bars in tension or compression, the lap coefficient varies from 1.0 to 1.5, according to the percentage of lapped bars relative to the total area of bars at the section considered, and transverse reinforcement is required at each end of the lap zone.

2.8.4 Anchorage Length

At both sides of any cross section, the reinforcement should be provided with an appropriate embedment length or other form of end anchorage. In earlier codes, it was also necessary to consider 'local bond' at sections where large changes of tensile force occur over short lengths of reinforcement, and this requirement remains in BS 5400.

The design bond stress fbd depends on the strength of the concrete, the type of bar and, in EC 2, the location of the bar within the concrete section during concreting. For example, the bond condition is classified as 'good' in the bottom 250 mm of any section, and in the top 300 mm of a section with 600 mm deep. In other locations, the bond condition is classified as 'poor'. Also in EC 2, the basic anchorage length, in tension, can be multiplied by several coefficients that take account of factors such as the bar shape, the cover and the effect of transverse reinforcement or pressure. For bars of diameter ≥40 mm, and bars grouped in pairs or bundles, additional considerations apply.

2.8.5 Shear

Suspended slab, beams and foundations are often subjected to large loads or reactions acting on small areas (Reynolds and Steedman, 2008). Shear in solid slabs under concentrated loads can result in punching failures on the inclined faces of truncated cones or pyramids. For design purposes, shear stresses are checked on given perimeters at specified distances from the edges of the loaded area. Where a load or reaction is eccentric with regard to a shear perimeter (e.g. at the edges of a slab, and in cases of moment transfer between a slab and a column), an allowance is made for the effect of the eccentricity. In cases where v exceeds vc, links, bent-up bars or other proprietary products may be provided in slabs not less than 200 mm deep.

In BS 8110, shear reinforcement is required to cater for the difference between the shear force and the shear resistance of the section without shear reinforcement. Equations are given for upright links based on concrete struts inclined at about 45^o, and for bent-up bars where the inclination of the concrete struts may be varied between specified limits. In BS 5400, a specified minimum amount of link reinforcement is required in addition to that needed to cater for the difference between the shear force and the shear resistance of the section without shear reinforcement. The forces in the inclined concrete struts are restricted indirectly by limiting the maximum value of the nominal shear stress to specified values.

In EC 2, shear reinforcement is required to cater for the entire shear force and the strength of the inclined concrete struts is checked explicitly. The inclination of the struts may be varied between specified limits for links as well as bent-up bars. In cases where upright links are combined with bent-up bars, the strut inclination needs to be the same for both.

2.8.6 Torsion

In normal beam-and-slab or framed construction, calculations for torsion are not usually necessary, adequate control of any torsional cracking in beams being provided by the required minimum shear reinforcement (Reynolds and Steedman, 2008). When it is judged necessary to include torsional stiffness in the analysis of a structure, or torsional resistance is vital for static equilibrium, members should be designed for the resulting torsional moment. The torsional resistance of a section may be calculated on the basis of a thin-walled closed section, in which equilibrium is satisfied by a closed plastic shear flow (Reynolds and Steedman, 2008). Solid sections may be modeled as equivalent thin-walled sections. Complex shapes may be divided into a series of sub-sections, each of which is modeled as an equivalent thin-walled section, and the total torsional resistance taken as the sum of the resistances of the individual elements. When torsion reinforcement is required, this should consist of rectangular closed links together with longitudinal reinforcement. Such reinforcement is additional to any requirements for shear and bending.

2.9 Reinforced Concrete Beams Design Tables and Charts

Design charts and tables are very useful tools for fast determining the percentage of reinforcement for reinforced concrete beams having known cross-sectional dimensions, characteristic strengths of the concrete and steel, and the ultimate design moment (Bayagoob, Yardim and Ramoda, 2013). They are also useful in the satisfaction of serviceability requirement (deflection and shear) for any structural members with varying cross section. This study will however focus on the development of design tables and charts relevant to design of reinforced concrete beams.

2.10 Review of Past Works

(Bayagoob, Yardim and Ramoda, 2013) conducted a study on development of design charts for rectangular section. Simplified design chart has been developed based on BS: 8110 – 97 design rules and several design examples were solved using the developed charts and tables.

(Nakov, 2012) developed the design charts for steel fiber reinforced concrete beams. Determination of the bearing capacity of the beams was made by using the Yield line theory. Different materials as well as different geometry were considered, making in total four hundred and five (405) combinations. All these combinations were a subject of design by the Yield line theory at the ultimate limit state and design charts with respect of some of the variables was developed.

The current study will however focus on formulation of simplified design tables and charts for the analysis and design of reinforced concrete beams.

CHAPTER THREE

3.0 MATERIALS AND METHODS

This chapter will present beams section and detailed procedure employed for the formulation of the design tables and charts. The design code of practice used for the formulation of the design tables and charts and also how the design charts can be read during design. Below is a presentation of the aforementioned information.

3.1 Beam Design Information

Two beams (both singly and doubly reinforced) will be considered for design using the developed design table and charts to be presented in subsequent chapter. The respective beams are to be designed to Euro code. Internal stresses (maximum bending moment and shear force) have been determined earlier using Euro code 2 (design of concrete structures). The beam will be designed using the design tables and charts developed for reinforced concrete beams with varying cross section. The design information is presented below:

Table 3.1 Design Information for the Reinforced Concrete Beams

Design Code	**Euro code 2: 1992 (design of concrete structures)**
General Loading Condition	**Specific Density of Concrete = 25KN/m^3** **Characteristic Strength of Concrete = 30N/mm^2** **Characteristic Strength of Steel = 500N/mm^2** **Concrete to reinforcement = 30mm**
Factor of Safety	**Dead Load (G_k) = 1.35** **Imposed Load (Q_k) = 1.5** **Ultimate design load = 1.35 G_k + 1.5 Q_k**
Design Data	**Fixed End Moment = $\frac{wl2}{12}$** **Free End Moment = $\frac{wl2}{8}$** **Minimum area of steel = 0.4%bh**

	For high yield steel = $\frac{0.15bh}{100}$
Beam Details (Singly Reinforced Concrete Beams)	**Width of beam is 250 and 300mm respectively.** **Overall depth of the beamsis 500 and 550mm respectively.** **Effective depth of the beams are 462 and 512mm respectively.**
Beam Details (Doubly Reinforced Concrete Beams)	**Width of beam is 275 and 300mm respectively.** **Overall depth of beam is 500 and 550mm respectively.** **Effective depth of the beams are 462 and 512mm respectively.**

3.2 Methods for Design of Reinforced Concrete Beams

The procedure for design of reinforced concrete beams according to euro code 2 (design of concrete structures) is presented below:

3.2.1 Ultimate Limit State Design

1. Determine the static system (simply supported beams, continuous beams, cantilever beams).
2. Choose the cross section (depth, width and span) for the beams.
3. Estimate the dead loads (self-weight of beam and it finishes, wall loads).

4 Estimate the live loads (load from superimposed structural elements like slabs decomposed into beams).
5 Analyze the structure and determine the internal stresses (maximum bending moment and shear force).
6 Determine the K-value and if the k-value is less than or equals 0.1569, design the beam for tension reinforcement only (for singly reinforced beams) but if the k-value exceeds 0.1569 design the beams for tension and compression reinforcements (for doubly reinforced beams).
7 Design the beams and determine the area of reinforcements.
8 Specify the number and diameter of the reinforcing bars.

3.2.2 Serviceability Limit State Checks

1 Check for deflection and where found inadequate, increase either the beam cross section (overall depth) or the area of reinforcing bars.
2 Check for shear stress and design for shear reinforcements.
3 Specify the diameter and spacing for links (stirrups).
4 Check for local bond and anchorage bond stress and where found inadequate, increase either the beam cross section (overall depth) of the beam.

3.3 Methods for Formulation of Design Tables and Charts for Reinforced Concrete Beams

Design tables and charts are valuable for the determination of the area of reinforcement at ultimate limit state design. The design tables and charts will be developed in accordance to Euro code 2 (Design of concrete structures). The design charts and table to be formulated based on relationship between K represented mathematically as $M/fcubd^2$ and mß represented as 100AsFy/bdFcu. Mathematical model will be derived for obtaining their relationship which will be used as basis for developing the reinforced concrete design charts and tables for both singly and doubly reinforced concrete beams. Below are procedures for obtaining the mathematical model.

3.3.1 Singly Reinforced Concrete Beams

Consider a rectangular section shown in Fig 3.0. In theory of reinforced concrete bending according to Euro code 2, the concrete is assumed to develop cracks in the region of tensile strain and after cracking, the tension is assumed to be carried by tensile reinforcement. The implication is that the plane section normal to the axis of the member remains plain after bending and the maximum strain in the concrete at the outermost fiber is 0.0035.

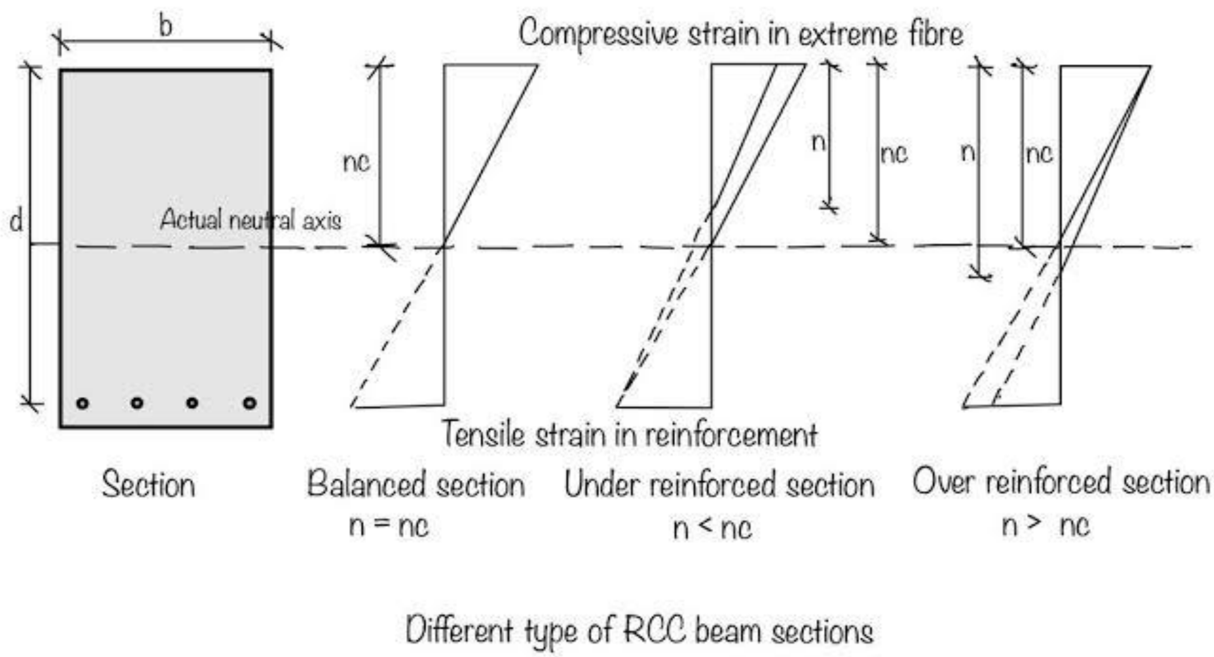


Fig 3.0: Stress to Strain Block Diagram for Singly Reinforced Rectangular Section

For a singly reinforced section, bending will give rise to resultant compressive strength (force), the equilibrium of the compressive and tensile equation can be written as:

$$0.95FyAs = 0.45Fcubx0.9x \qquad (3.0)$$

Thus the depth of the uniform stress block of the compression concrete (x) is obtained from the above equation

$x = 2.346AsFy/Fcu\, b$ (3.1)

The lever arm of the resultant concrete force about the tension steel can be expressed as:

$z = d - 0.45x$ (3.2)

Substitute equation (3.1) into equation (3.2) we obtain the relation:

$z = d\,[1 - 1.056AsFy/Fcu\, bd]$ (3.3)

Taking the moment about the resultant concrete compression force, the ultimate moment of resistance of the section can be expressed as:

$Mu = 0.95FyAsZ$ (3.4)

Substitute equation (3.3) into equation (3.4) we obtain the relation:

$Mu = 0.95FyAsd\,[1 - 1.056AsFyFcu\, bd]$ (3.5)

Dividing both terms of equation (3.5) by $Fcubd^2$ and using the notations:

$K = Mu/Fcubd2ß = 100As/bd$ and $m = Fy/Fcu$We obtain the following mathematical model:

$K = 0.0095mß - 0.0001(mß)2$ (3.6)

Euro code 2: (1992) code stipulates that the lever arm for singly reinforced concrete beams must not exceed 0.95d in order to offer a reasonable concrete area in compression.

Relationship between K and Z is represented by the relation:

$z/d - (z/d)2 = k/0.9$ (3.7)

At limit of Z=0.95d, therefore the value of k is less than or equal to 0.0428, the steel area should be calculated using Z = 0.95d. Substituting Z = 0.95d into equation (3.4) and using the earlier stated notations (m, ß) obtain the following is obtained:

$K = 0.009m\beta$ for K less than or equal to 0.00428 (3.8)

$K = 0.0095m\beta - 0.0001(m\beta)2$ (3.9)

Therefore, for the purpose of application in the design of singly reinforced concrete beams, equation 3.8 will be redefined as follows:

$M\beta = 100AsFy/bdFcu.$ (3.10)

Substituting Equation (3.10) into Equation (3.8) we have:

$$\boldsymbol{K = 0.009\left(\frac{100AsFy}{bdFcu}\right)} \quad (3.11)$$

Equation (3.11) is a mathematical model used for formulating the design charts and tables for singly reinforced concrete beams, if the value of K is known, the value of Area of steel (As)can be determined and Area of steel required can be specified.

3.3.2 Doubly Reinforced Concrete Beams

Doubly reinforced concrete section is applicable when the applied moment is greater than the ultimate moment of resistance (Euro code 2:1992). The moment resistance of a doubly reinforced section is shown in Figure 3.2 and it is given by the following:

$Mu = K1Fcubd2 + 0.95FyAs1\ (d - d1)$ (3.12)

Where K is a factor depending on the ratio of the stress block depth to the effective depth (x/d). Dividing both side of the equation by the term $Fcubd^2$ and using the notation $\beta 1 = 100As1/bd$in addition to the notation used earlier we will obtain the following relation:

$K = K1 + 0.0095m\beta 1\ (d - d1)$ (3.13)

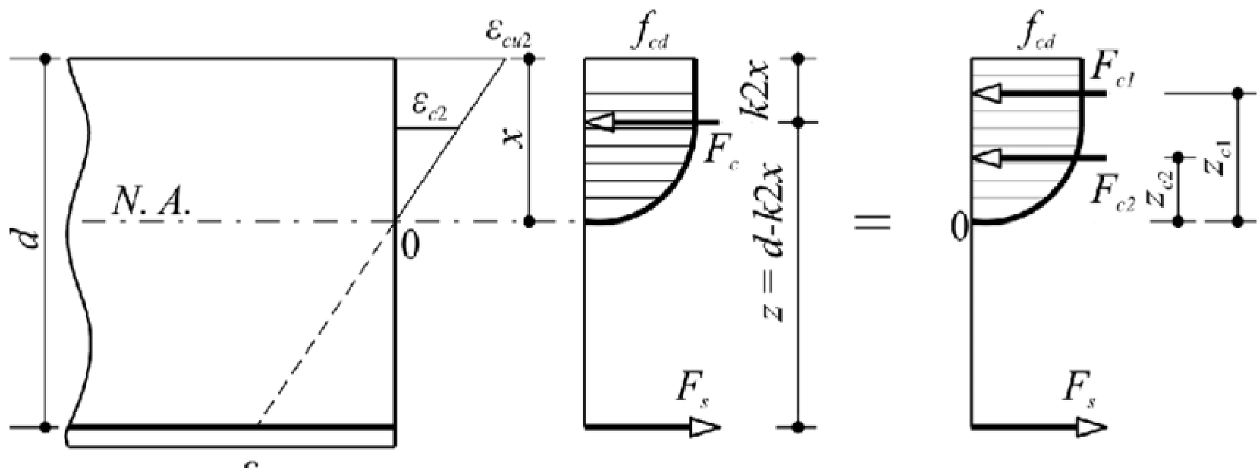


Fig 3.1: Stress – Strain Curve for Doubly Reinforced Concrete Beams (Euro code 2: 1992).

Separate charts are required for different values of d^1. If a value of 0.1d is used for the effective depth of the compression equation (3.13) becomes:

$K = K1 + 0.00855mß1$ (3.14)

Reference to Fig 3.1, the equilibrium of compressive and tensile forces as stated in Euro code 2(1992) can be expressed as:

$0.95FyAs = 0.45Fcub0.9x + 0.95FyAs1$ (3.15)

Depending on the moment distribution factor (ßb), the neutral axis depth can be expressed as a fraction of the effective depth (c) as:

$x = cd$ (3.16)

Substituting equation (3.16) into equation (3.15) we obtain the following relation:

$0.95FyAs = 0.45Fcu\, b\, c\, d + 0.95FyAs1$ (3.17)

Dividing both terms of the above equation by the term 0.95Fybd and using the earlier stated notations (ß $ß^1$) and m, the percentage of the compression steel $ß^1$ can be expressed as:

$ß^1 = ß - 42.6c/m$ (3.18)

Substituting equation (3.18) into equation (3.14) we obtain the following:

$K = (K1 - 0.365c) + 0.00855mß$ or $K = ß + 0.00855mß$ (3.19)

Applying equation of line $(y = mx + c)$, it can be deduced that from the comparison of equation (3.14) and equation (3.19) it is clearly be clearly observed that the slope (0.000855) are the same and only the intercept are different, thus the implication is that the same curve can be used for the determination of area of reinforcement at both the tension and compression zone. However, this study will present both separate and combined charts and tables for the design of both singly and doubly reinforced beams.

However, if the neutral axis depth (x) is taken as 0.5d, equation (3.14) becomes:

$$K = 0.156 + 0.00855mß. \qquad (3.20)$$

While equation (3.19) becomes;

$$K = -0.02625 + 0.00855mß \qquad (3.21)$$

However, for design of doubly reinforced concrete beam equation 3.20 can be redefined as:

$$Mß = 100AsFy/bdFcu. \qquad (3.22)$$

Substituting equation (3.22) into equation (3.20) we have:

$$K = 0.156 + 0.00855\left(\frac{100AsFy}{bdFcu}\right). \qquad (3.23)$$

Equation (3.23) will be applicable in the formulation of design tables and charts for doubly reinforced concrete beams. If the value of K is known, then the area of steel required can be specified.

The design charts and tables will be formulated for singly reinforced concrete beams, doubly reinforced concrete beams and both single and doubly reinforced concrete beams. For formulation of design tables and charts for both singly and doubly reinforced concrete beams (combined), the tensile steel area (As) will be read from the lower abscissa (x- axis) while the compression steel area will be read from the upper abscissa (x –axis). For the sake of clarity during interpretation of both tension and compression steel area for the developed design charts for both singly and doubly reinforced concrete beams, it is however important that the starting point of the compression steel area located at the upper abscissa (x-axis) must undergo a shift. The shifting value of the compression steel area is dependent on the K^1 and the intercept. The K^1 in turn depends on the moment redistribution factor (ßb) and the neutral axis depth (x). Below is a range of shifting values for the compression steel and how they were obtained.

Moment Redistribution Factor (ßb)	Neutral Axis Depth (x)	K^1	Intercept (ß)	Shifting Value
0.9	0.5d	0.156	-0.02625	21.32
0.8	0.4d	0.132	-0.0138	17.05
0.7	0.3d	0.109	-0.00435	13.26

Column (1 and 2) was determined based on reduction of 10% (0.1) from initial values.

Column (3) wasfrom equation (3.14) and subsequent values were obtained by 15% reduction of the initial value as specified by the code.

Column (4) was obtained using equation (3.19)

Column (5) which is the shifting value was obtained using this formula:

Shifting Value = (K^1 + intercept)/Slope

Equation (3.11 and 3.23) will be valuable for the formulation of the design tables and charts for both singly and doubly reinforced concrete beams.

Equation (3.14 and 3.19) will be valuable for obtaining the shifting value for interpretation of the compression steel area.

The design charts and tables and their method of application during design of both singly and doubly reinforced concrete beams will be presented in chapter 4 of the analytical study.

CHAPTER FOUR

RESULTS AND DISCUSSION

Key findings gleaned from the analytical study are presented in this section. Some of the findings are formulated design table for specification of area of reinforcement required at ultimate limit state for both singly and doubly reinforced concrete beams, formulated design charts for specification of area of reinforcement required at ultimate limit state of design for singly and doubly reinforced concrete beams, developed consolidated (charts for both singly and doubly reinforced concrete beams) for specification of area of reinforcement required, method of application of the developed design tables and charts and serviceability limit checks for accuracy of developed design tables and charts. Below is a presentation of the aforementioned information.

4.1 Results

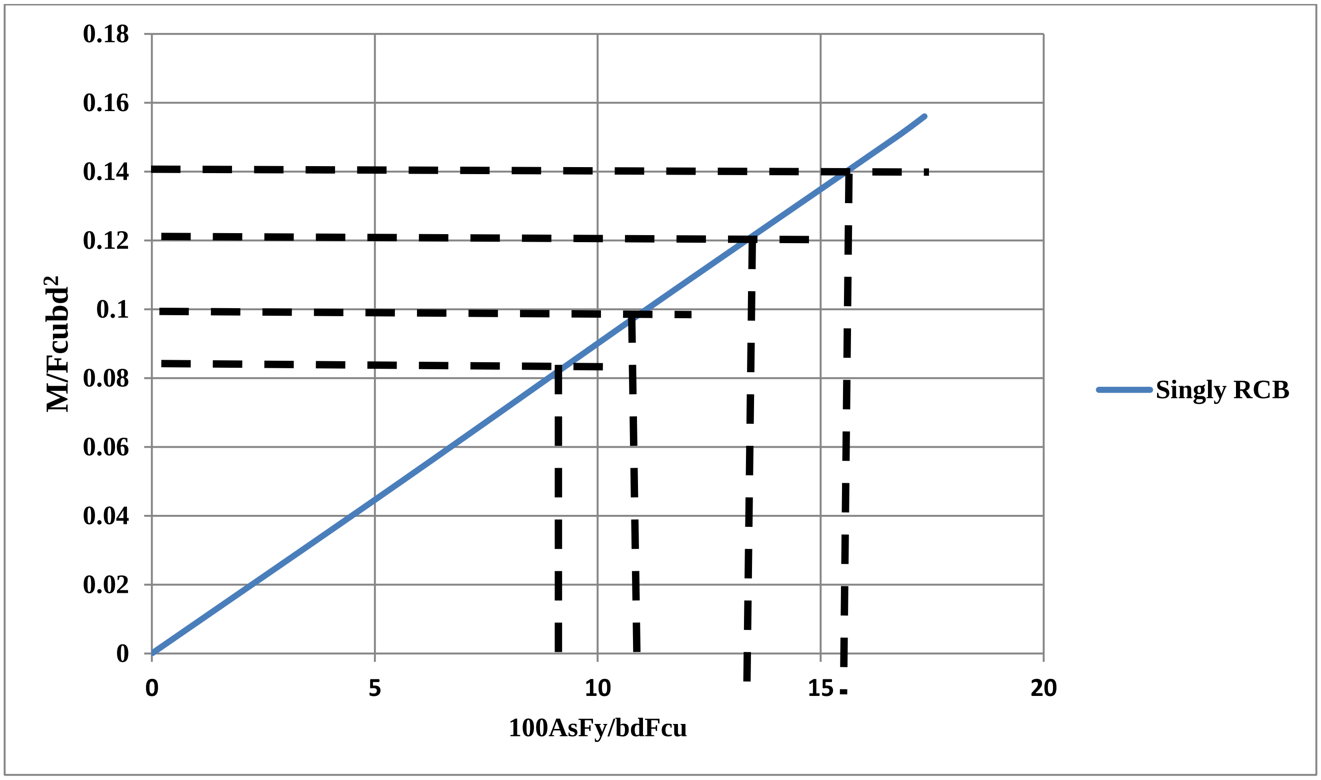


Figure 4.1: Developed Design Chart for Singly Reinforced Concrete Beam (Developed using equation 3.11 generated in chapter three).

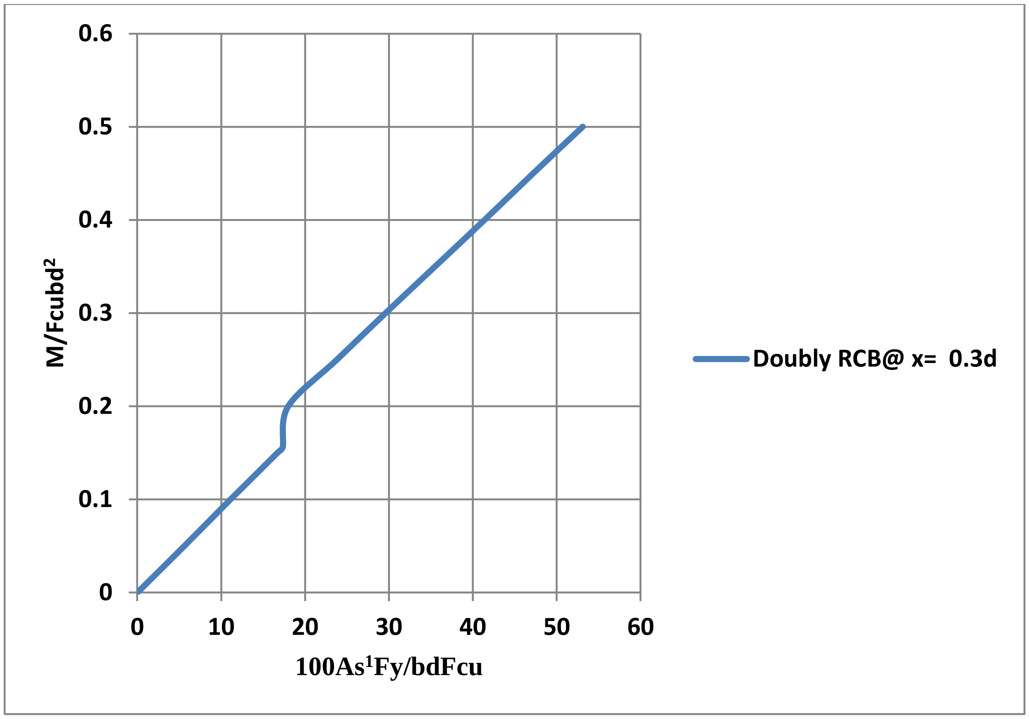


Figure 4.2: Developed Design Chart for Doubly Reinforced Concrete Beam at Neutral Axis Depth of 0.3d and Moment Redistribution factor of 0.7 (Developed using equation 3.23 generated in chapter three).

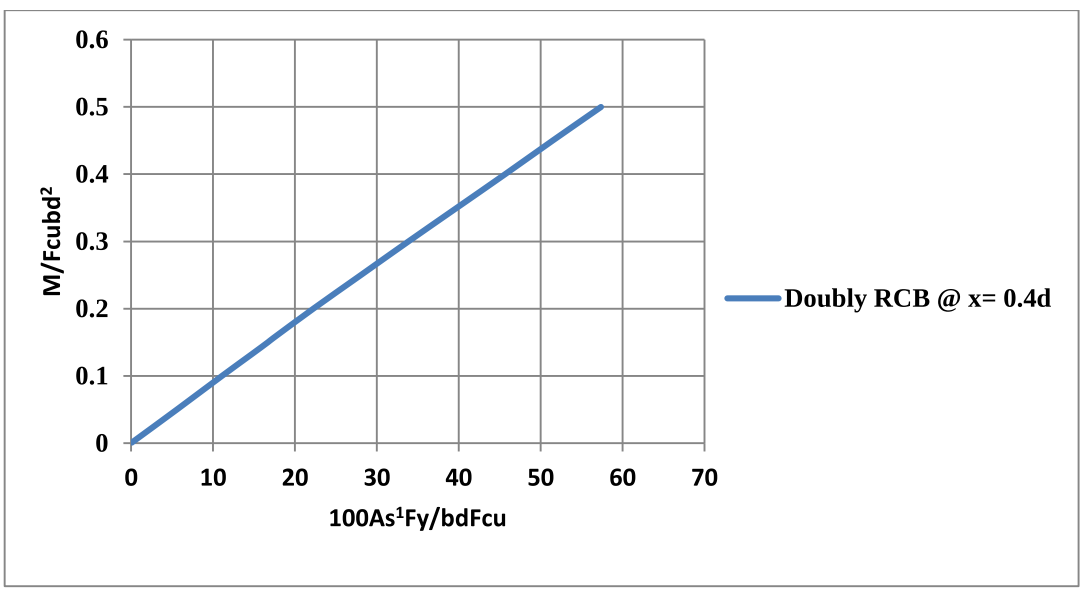


Figure 4.3: Developed Design Chart for Doubly Reinforced Concrete Beam at Neutral Axis Depth of 0.4d and Moment Redistribution factor of 0.8 (Developed using equation 3.23 generated in chapter three).

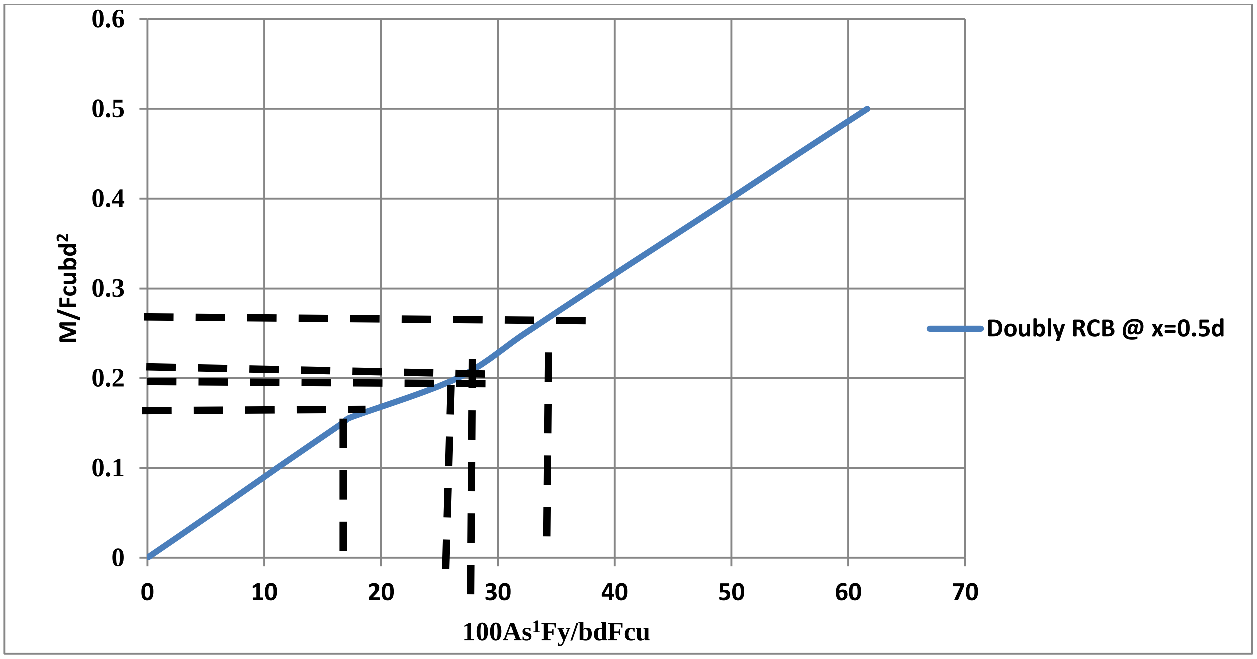


Figure 4.4: Developed Design Chart for Doubly Reinforced Concrete Beam at Neutral Axis Depth of 0.5d and Moment Redistribution factor of 0.9 (Developed using equation 3.23 generated in chapter three).

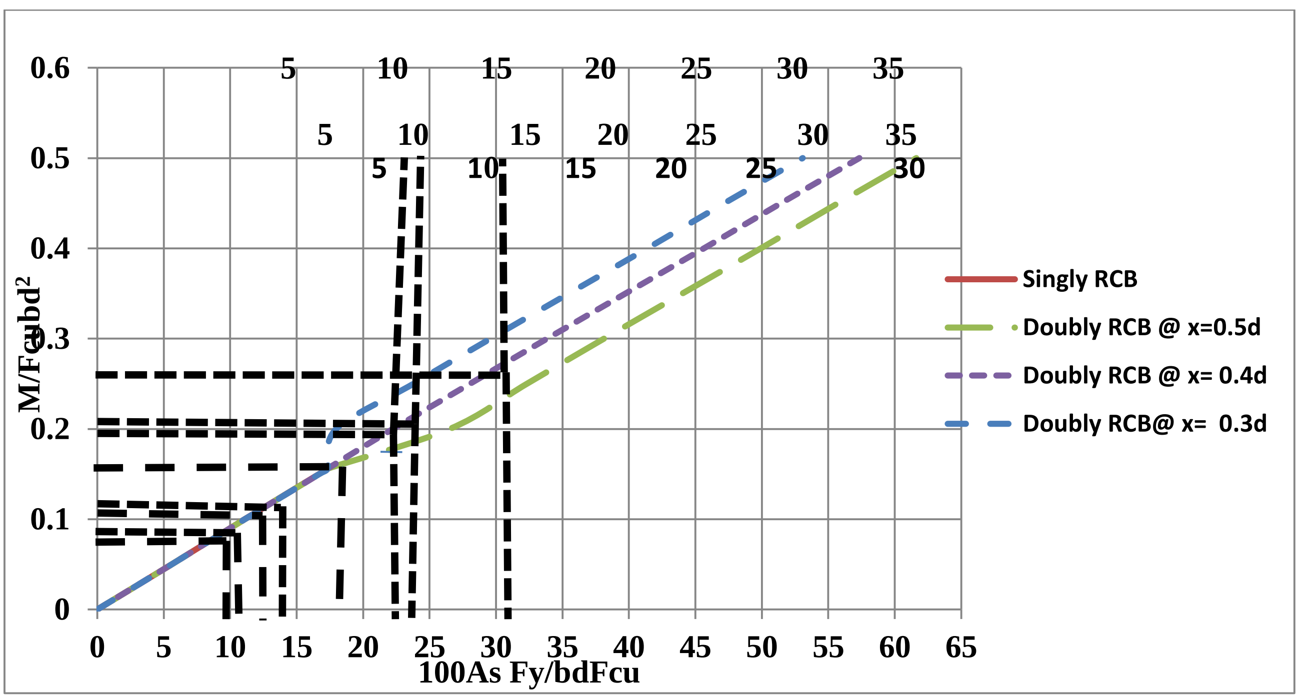


Figure 4.5: Developed Consolidated Design Charts for both Singly and Doubly Reinforced Concrete Beam (Developed using equation 3.23 generated in chapter three).

Table 4.1: Value for Area of Steel Required (mm^2) at Steel grade and Concrete grade of 410N/mm^2 and 25N/mm^2 and Redistribution factor of 0.9, the width of the beam is 250mm (Developed using equation 3.11 and 3.23 generated in chapter three).

M/fcubd²	**100AsFy/Fcubd**	**Effective Depth (mm)**		
		450	500	550
0.05	5.6	384	427	470
0.1	11.1	762	846	931
0.15	16.7	1146	1273	1400
0.2	26.52	1819	2021	2223
0.25	32.32	2217	2463	2710
0.3	38.12	2615	2905	3196
0.35	44.02	3020	3355	3691
0.4	49.92	3424	3805	4185
0.45	55.72	3822	4247	4672
0.5	61.62	4227	4697	5166

Table 4.1 was formulated using the relation

$$\boldsymbol{K = 0.009\left(\frac{100AsFy}{bdFcu}\right).} \quad \textbf{(3.11)}$$

$$\frac{100AsFy}{bdFcu} = \frac{k}{0.009}$$

For k = 0.05 we have

$$\frac{100AsFy}{bdFcu} = \frac{0.05}{0.009} = 5.6$$

$$\frac{100AsFy}{bdFcu}. = 5.6$$

Making As subject of the formula and substituting the values of fy = 410N/mm^2,fcu = 25N/mm^2, d = 450mm, b = 250mm

We have

As = $\frac{5.6x250x450x25}{100x410}$ = 384mm

Table 4.2: Value for Area of Steel Required (mm^2) at Steel grade and Concrete grade of 410N/mm^2 and 25N/mm^2 and Redistribution factor of 0.8, the width of the beam is 250mm (Developed using equation 3.11 and 3.23 generated in chapter three).

M/fcubd^2	**100AsFy/Fcubd**	**Effective Depth (mm)**		
		450	500	550
0.05	5.6	384	427	470
0.1	11.1	762	846	931
0.15	16.7	1146	1273	1400
0.2	22.25	1526	1696	1865
0.25	28.05	1924	2138	2352
0.3	33.85	2322	2580	2838
0.35	39.75	2727	3030	3333
0.4	45.65	3131	3479	3827
0.45	51.45	3529	3921	4313
0.5	57.35	3934	4371	4808

Table 4.2 was formulated using the relation

$$\boldsymbol{K = 0.009\left(\frac{100AsFy}{bdFcu}\right).}$$

$$\frac{100AsFy}{bdFcu} = \frac{k}{0.009}$$

For k = 0.1we have

$$\frac{100AsFy}{bdFcu} = \frac{0.1}{0.009} = 11.1$$

$$\frac{100AsFy}{bdFcu}. = 11.1$$

Making As subject of the formula and substituting the values of fy = 410N/mm^2, fcu = 25N/mm^2, d = 450mm, b = 250mm

We have

As = $\frac{11.1x250x450x25}{100x410}$ = 762mm

Table 4.3: Value for Area of Steel Required (mm^2) at Steel grade and Concrete grade of 410N/mm^2 and 25N/mm^2 and Redistribution factor of 0.7, the width of the beam is 250mm (Developed using equation 3.11 and 3.23 generated in chapter three).

M/fcubd^2	**100AsFy/bdFcu**	**Effective Depth (mm)**		
		450	500	550
0.05	5.6	384	427	470
0.1	11.1	762	846	931
0.15	16.7	1146	1273	1400
0.2	17.99	1234	1371	1508
0.25	23.79	1632	1813	1995
0.3	29.59	2030	2255	2481
0.35	35.49	2435	2705	2976
0.4	41.39	2839	3154	3470
0.45	47.19	3237	3597	3950
0.5	53.09	3642	4046	4451

Table 4.4: Formulated Design Charts for Singly Reinforced Concrete Beam and Comparison with Reinforced Concrete Beam Design to Euro code

Design Examples	**Charts Solution (mm^2)**	**Euro code 2 Solution (mm^2)**	**Error (%)**
1 (Support)	839	949	11.6
2 (Span)	519	522	0.6

3	1011	1259	19.7
4	880	1024	14.1
5	1153	1371	15.9
6	1031	1127	8.5

This comparison in table 4.4 was obtained using result from the design information presented below, the area of steel was solved using formulated charts and tables and euro code solution

Table 4.5: Formulated Design Table for Singly Reinforced Concrete Beam and Comparison with Reinforced Concrete Beam Design to Euro code

Design Examples	**Table Solution (mm^2)**	**Euro code 2 Solution (mm^2)**	**Error (%)**
1 (Support)	960	949	1.2
2 (Span)	560	522	7.3
3	980	1259	22.1
4	880	1024	16.4
5	1185	1371	13.6
6	984	1127	12.7

Table 4.6: Formulated Design Charts for Doubly Reinforced Concrete Beam and Comparison with Reinforced Concrete Beam Design to Euro code

Design Examples	**Charts Solution (mm^2)**		**Euro code 2 Solution (mm^2)**		**Error (%)**	
	Tension area	**Compression area**	**Tension area**	**Compression area**	**Tension area**	**Compression area**
1	1979	513	2012	369	1.6	39.0
2	1529	93	1829	65	16.4	43.1
3	2335	826	2472	824	5.5	0.24
4	2144	536	2301	471	6.8	13.8

Table 4.7: Formulated Design Table for Doubly Reinforced Concrete Beams and Comparison with Reinforced Concrete Beam Design to Euro code

Design Examples	Table Solution for Tension area (mm^2)	Euro code 2 Solution for Tension (mm^2)	Error (%)
1	1912	2012	5.0
2	1485	1829	18.8
3	2335	2472	5.5
4	2144	2301	6.8

The formulated design tables and charts for singly and doubly reinforced concrete beams was based on a mathematical derivation developed in Chapter three (equation 3.11 and 3.23). The design charts and tables were developed in accordance reinforced concrete design code of practice as specified by Euro code 2 (design of reinforced concrete structures). The developed design tables display the values of area of steel required at ultimate limit state of design for both singly and doubly reinforced concrete beams. The table shows the value of area of steel required at varying values of effective depth and moment redistribution factor. If the value of K ($\frac{M}{Fcubd2)}$) and the effective depth is known, the area of steel required at ultimate limit state of design can be specified.

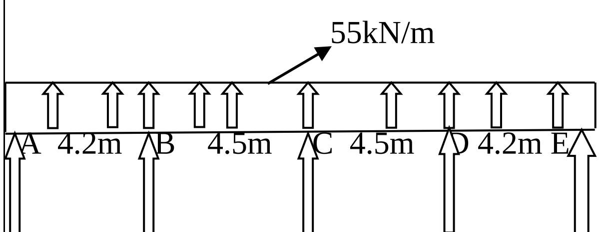
The developed design charts is a graphical representation of K ($\frac{M}{Fcubd2)}$) against mß($\frac{100AsFy}{bdFcu}$). The value of K is plotted at the ordinate (Y-axis) while the value of mß is plotted at the abscissa (X-axis). If the value of K is known, the value of mß ($\frac{100AsFy}{bdFcu}$) can be read from the formulated design charts and thereafter, area of steel required at ultimate limit state of design can be specified. The consolidated (both singly and doubly reinforced concrete beam) design charts shows the values of mß($\frac{100AsFy}{bdFcu}$) at both the lower and upper abscissa (X-axis). The area of tension reinforcement can be read from the lower abscissa while the area of compression reinforcement can be read from the upper abscissa. This chart is applicable for both singly and doubly reinforced concrete beam. Either the developed design tables or charts can be used for design of both singly or doubly reinforced concrete beams at ultimate limit state.

4.2 Design Examples

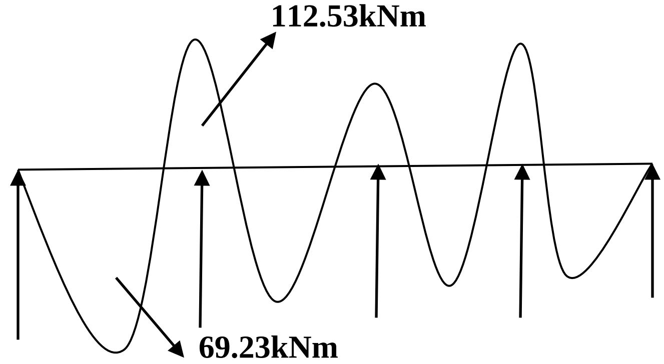
In order to evaluate the validity of the developed design tables and charts, design examples for two reinforced concrete beams with varying cross section have been solved using the developed design tables and charts and design results were compared to that obtained using Euro code 2 design code of practice. The design parameters are presented below:

4.2.1 Practical Design of Reinforced Concrete Beam

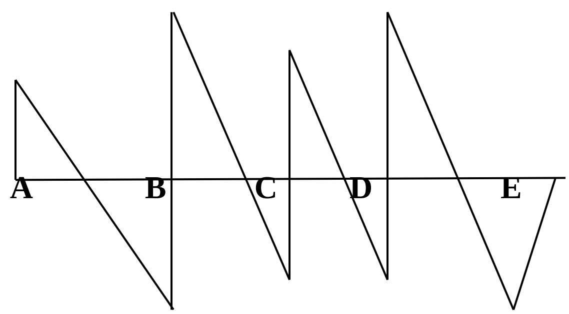
Design Information		
	Design Stresses	Steel Fy= 410N/mm^2, Concrete Fcu = 25N/mm^2
	Relevant Codes	Euro code 2, Part 1: 1992
	Exposure Condition	Mild for all element Cover to reinforcement = 25mm Breadth of beam =250mm Overall depth of beam = 450mm
	Design Data	K = $\frac{M}{Fcubd2}$, As = $\frac{M}{0.87FyZ}$, As^1 = $\frac{(K-Ku)Fcubd2}{0.87Fy(d-d1)}$

Member Reference	Calculation	Output
Table 4.2, Euro code 2: 1992	A four span continuous beam shown in the figure below support the following loads, itown self weight and finishes inclusive. Using Fcu = 25N/mm^2, Fy = 410N/mm^2, design the beam completely to Euro code. Assume width of the beam to be 230mm and flange width of 780mm (Tee beam). Load Assemblage Beam own load = 5kN/m Load from wall and roof = 28.2kN/m Load on slab = 21kN/m Ultimate design load = 5kN/m + 28.2kN/m + 21kN/m = 54.2kN/m say 55kN/m  Applying the ultimate load we have the loading as shown in the figure above and by symmetry MA = ME = 0.00 Also by similar reasoning, MB = MD =?kNm, the value of this moment and Mc can be obtained using Clapeyrons three moments equation. Applying Clapeyrons three moment equation we have: Span AB and BC	55kN/m

	4.2MA + 2MB (4.2+ 4.5) + 4.5MC = 55/4 (4.2^3 + 4.5^3) MA = ME = 0.00 17.4MB + 4.5MC = 2354 (1) Similarly, 4.5MB + 18MC + 4.5MD= 2597 (2) 4.5MC + 17.4MD = 2354 (3) Solving Simultaneously we have: MB = MD = 112.525kNm and MC = 88.015kNm Free Moments at mid spans: Span A-B and D-E = 0.125 (4.2^2)55 = 125.685kNm Span B- C and C-D = 0.125 (4.5^2)55 = 144.281kNm Using the principle of interpolation and assuming that the maximum moment occurs at the mid span the span moment are calculated as follows: Span A-B = M^o- $MA + \frac{MB}{2}$ Span A-B = 125.685 – (0+112.525)/2 = 69.423kNm Span B-C = 144.281 – (112.525+ 88.015)/2 = 44.011kNm Span C-D = 144.281 – (112.525 + 88.015)/2 = 44.011kNm Computation of Shear Force VAB = 55x4.2/2 + 0-112.525/4.2 = 92.9kN VBA = 55X4.2/2 + 112.525-0/4.2 = 146.5kN VBC = 55x4.5/2 + 112.525-88.015/4.5 = 133.7kN VCB = 55X4.5/2 + 88.015-112.525/4.5 = 122.8kN	
Table 10.2 Pp-86, Euro code 2: 1992	VCB = VCD = 122.8kN VDC = VBC = 133.7kN VDE = VBA = 146.5kN VBD = VAB = 929kN	



Bending Moment Diagram



Design of Beam Using Developed Design Chart and Tables

Support Moment

Moment = 112.5kNm

Effective depth (d) = 450-25-20 = 405mm

$$K = \frac{M}{Fcubd2} = 112.5x\frac{106}{25\ x\ 250x405x405} =$$

0.109<0.1569

Therefore, only tension reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.4

	above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.109) is 13.6 Therefore $\frac{100AsFy}{bdFcu}$ = 13.6, determining the area of steel required from the relation: As = $\frac{13.6x250x405x25}{100x410}$ = 839mm^2 Therefore, area of steel required for beam section having a moment of 112.5kNm and overall depth and width of 450 and 250mm is 839mm^2 Area of steel provided is 5Y16 (As = 1010mm^2) Using the formulated design table for moment redistribution factor of 0.9, K value of 0.109 and effective depth of 405mm, area of steel required is 960mm^2 Therefore, area of steel provided is 5Y16 (As = 1010mm^2) **Span Moment** Moment = 69.4kNm Effective depth (d) = 450-25-20 = 405mm K = $\frac{M}{Fcubd2}$ = $69.4x \frac{106}{25\ x\ 250x405x405}$ = 0.067<0.1569 Therefore, only tension reinforcement is required. Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.4 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.067) is 8.4 Therefore $\frac{100AsFy}{bdFcu}$ = 8.4, determining the area of steel required from the relation: As = $\frac{8.4x250x405x25}{100x410}$ = 519mm^2 Therefore, area of steel required for beam section having a moment of 69.4kNm and overall depth and	

width of 450 and 250mm is 519mm^2

Area of steel provided is 4Y16 (As = 804mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.067 and effective depth of 405mm, area of steel required is 520mm^2

Therefore, area of steel provided is 4Y16 (As = 804mm^2)

Design Using Euro code 2 Code of Practice

Support Moment

Moment = 112.5kNm

Effective depth (d) = 500-20-25 = 405mm

K = $\frac{M}{Fcubd2}$ = $112.5x\frac{106}{25\ x\ 250x405x405}$ = 0.109<0.1569

Therefore, only tension reinforcement is required.

From Table 10.2, design of reinforced concrete beam to Euro code, lever arm (la) for K value of 0.12 = 0.82

Therefore As = $\frac{112.5x106}{0.87x410x0.82x405}$ = 949mm^2

Provide 5Y16 (As = 1010mm^2)

Design of Span Moment

Moment = 69.4kNm

Effective depth (d) = 500-20-25 = 405mm

K = $\frac{M}{Fcubd2}$ = $69.4x\frac{106}{25\ x\ 250x405x405}$ = 0.067<0.1569

Therefore, only tension reinforcement is required.

From Table 10.2, design of reinforced concrete beam to Euro code, lever arm (la) for K value of 0.12 = 0.92

Therefore As = $\frac{69.4x106}{0.87x410x0.92x405}$ = 522mm^2

	Provide 4Y16 (As = 804mm^2)	

4.3 Other Design Examples

4.3.1 Singly Reinforced Section

Design moments for singly reinforced concrete beam = 170kNm and 185kNm respectively

Design moment for doubly reinforced concrete beams = 275kNm and 350kNm respectively.

Width of beam = 250mm and 300mm respectively.

Overall depth of beam = 500 and 550mm respectively.

Cover to reinforcement = 25mm

4.3.1.1 Design of Singly Reinforced Concrete Beams Using Formulated Design Tables and Charts.

Beam Section with Moment of 170kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ = $170x \frac{106}{25\ x\ 250x467x467}$ = 0.12<0.1569

Therefore, only tension reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.4 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.12) is 14.2.

Therefore $\frac{100AsFy}{bdFcu}$ = 14.2, determining the area of steel required from the relation:

As = $\frac{14.2x250x467x25}{100x410}$ = 1011mm^2

Therefore, area of steel required for beam section having a moment of 170kNm and overall depth and width of 500 and 250mm is 1011mm^2

Area of steel provided is 6Y16 (As = 1210mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.12 and effective depth of 467mm, area of steel required is 980mm^2

Therefore, area of steel provided is 5Y16 (As = 1010mm^2)

Beam Section with Moment of 170kNm, Overall depth of 550mm, Width of 300mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ = $170x\frac{106}{25\ x\ 300x517x517}$ = 0.08<0.1569

Therefore, only tension reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.4 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.08) is 9.3.

Therefore $\frac{100AsFy}{bdFcu}$ =9.3, determining the area of steel required from the relation:

As = $\frac{9.3x300x517x25}{100x410}$ = 880mm^2

Therefore, area of steel required for beam section having a moment of 170kNm and overall depth and width of 550 and 300mm is 880mm^2

Area of steel provided is 8Y12 (As = 905mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.08 and effective depth of 517mm, area of steel required is 880mm^2

Therefore, area of steel provided is 8Y12 (As = 905mm^2)

Beam Section with Moment of 185kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ = $185x\frac{106}{25\,x\,250x467x467}$ = 0.14 <0.1569

Therefore, only tension reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.4 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.14) is 16.2

Therefore $\frac{100AsFy}{bdFcu}$ = 16.2, determining the area of steel required from the relation:

As = $\frac{16.2x250x467x25}{100x410}$ = 1153mm^2

Therefore area of steel required for beam section having a moment of 185kNm and overall depth and width of 500 and 300mm is 1153mm^2

Area of steel provided is 6Y16 (As = 1210mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.14 and effective depth of 467mm, area of steel required is 1185mm^2

Therefore, area of steel provided is 6Y16 (As = 1210mm^2)

Beam Section with Moment of 185kNm, Overall depth of 550mm, Width of 300mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ = $185x\frac{106}{25\,x\,300x517x517}$ = 0.09 <0.1569

Therefore, only tension reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.4 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.09) is 10.9

Therefore $\frac{100AsFy}{bdFcu}$ = 10.9, determining the area of steel required from the relation:

As = $\frac{10.9x300x517x25}{100x410}$ = 1031mm^2

Therefore area of steel required for beam section having a moment of 185kNm and overall depth and width of 550 and 300mm is 1031mm^2

Area of steel provided is 6Y12 (As = 1210mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.09 and effective depth of 517mm, area of steel required is 984mm^2

Therefore, area of steel provided is 5Y16 (As = 1010mm^2).

4.4 Design of Doubly Reinforced Concrete Beams

4.4.1 Design of Doubly Reinforced Concrete Beams Using Formulated Design Tables and Charts.

Beam Section with Moment of 275kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ $= 275x \frac{106}{25\ x\ 250x467x467}$ = 0.2 >0.1569

Thereforetension and compression reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.5 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.2) are 27.8 read from the lower abscissa and 7.2 read from the upper abscissa (x-axis).

Therefore $\frac{100AsFy}{bdFcu}$ = 27.8, determining the area of tension reinforcement required.

As = $\frac{27.8x250x467x25}{100x410}$ = 1979mm^2

Therefore area of tension steel required for beam section having a moment of 275kNm and overall depth and width of 500 and 250mm is 1979mm^2

Area of steel provided is 7Y20 (As = 2200mm^2)

Determining area of Compression Reinforcement

$\frac{100AsFy}{bdFcu}$ = 7.2 obtained at upper abscissa from Figure 4.5 above.

As = $\frac{7.2x250x467x25}{100x410}$ = 513mm^2

Therefore area of compression steel required for beam section having a moment of 275kNm and overall depth and width of 500 and 250mm is 494mm^2

Area of steel provided is 7Y20 (As = 566mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.21 and effective depth of 467mm, area of steel required as shown in Table 4.4 above is 1912mm^2

Therefore, area of steel provided is 7Y20 (As = 2200mm^2)

Beam Section with Moment of 275kNm, Overall depth of 550mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ = $275x\,\frac{106}{25\,x\,250x517x517}$ = 0.16>0.1569

Therefore tension and compression reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.5 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.16) are 19.4 read from the lower abscissa and 1.3 read from the upper abscissa (x-axis).

Therefore $\frac{100AsFy}{bdFcu}$ = 19.4 determining the area of tension reinforcement required.

As = $\frac{19.4x250x517x25}{100x410}$ = 1529mm^2

Therefore area of tension steel required for beam section having a moment of 275kNm and overall depth and width of 550 and 250mm is 1356mm^2

Area of steel provided is 7Y16 (As = 1410mm^2)

Determining area of Compression Reinforcement

$\frac{100AsFy}{bdFcu}$ = 1.3, obtained at upper abscissa from Figure 4.5 above.

As = $\frac{1.3x250x467x25}{100x410}$ = 93mm^2

Therefore area of compression steel required for beam section having a moment of 275kNm and overall depth and width of 550 and 250mm is 426mm^2

Area of steel provided is 2Y10 (As = 566mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.16 and effective depth of 517mm, area of steel required as shown in Table 4.4 above is 1485mm^2

Therefore, area of steel provided is 5Y20 (As = 1570mm^2)

Beam Section with Moment of 350kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ = $350x\frac{106}{25\ x\ 250x467x467}$ = 0.26 >0.1569

Therefore tension and compression reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.5 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.26) are 32.8 read from the lower abscissa and 11.6 read from the upper abscissa (x-axis).

Determining the area of tension reinforcement required.

Therefore $\frac{100AsFy}{bdFcu}$ = 32.8

As = $\frac{32.8x250x467x25}{100x410}$ = 2335mm^2

Therefore area of tension steel required for beam section having a moment of 350kNm and overall depth and width of 500 and 250mm is 2335mm^2

Area of steel provided is 8Y20 (As = 2510mm^2)

Determining area of Compression Reinforcement

$\frac{100AsFy}{bdFcu}$ = 11.6 obtained at upper abscissa from Figure 4.5 above.

As = $\frac{11.6x250x467x25}{100x410}$ = 826mm^2

Therefore area of compression steel required for beam section having a moment of 350kNm and overall depth and width of 500 and 250mm is 826mm^2

Area of steel provided is 5Y16 (As = 1010mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.26 and effective depth of 467mm, area of steel required as shown in Table 4.4 above is 2335mm^2

Therefore, area of steel provided is 8Y20 (As = 2510mm^2)

Beam Section with Moment of 350kNm, Overall depth of 550mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ $= 350x \frac{106}{25\ x\ 250x517x517}$ = 0.21>0.1569

Therefore, tension and compression reinforcement is required.

Using the developed design charts for moment redistribution factor of 0.9 as shown in Figure 4.5 above, the value of $\frac{100AsFy}{bdFcu}$ corresponding to the value K (0.21) are 27.2 read from the lower abscissa and 6.8 read from the upper abscissa (x-axis).

Determining area of tension reinforcement required.

Therefore $\frac{100AsFy}{bdFcu}$ = 27.2

As = $\frac{27.2x250x517x25}{100x410}$ = 2144mm^2

Therefore area of tension steel required for beam section having a moment of 350kNm and overall depth and width of 550 and 250mm is 2144mm^2

Area of steel provided is 8Y20 (As = 2510mm^2)

Determining area of Compression Reinforcement

$\frac{100AsFy}{bdFcu}$ = 6.8 obtained at upper abscissa from Figure 4.5 above.

As = $\frac{6.8x250x517x25}{100x410}$ = 536mm^2

Therefore area of compression steel required for beam section having a moment of 350kNm and overall depth and width of 550 and 250mm is 536mm^2

Area of steel provided is 4Y16 (As = 804mm^2)

Using the formulated design table for moment redistribution factor of 0.9, K value of 0.21 and effective depth of 517mm, area of steel required as shown in Table 4.4 above is 2144mm^2

Therefore, area of steel provided is 8Y20 (As = 2510mm^2)

4.5 Design of Singly Reinforced Concrete Beams Using Code of Practice (Euro code 2: 1992)

Beam Section with Moment of 170kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ = $170x \frac{106}{25\ x\ 250x467x467}$ = 0.12<0.1569

Therefore, only tension reinforcement is required.

From Table 10.2, design of reinforced concrete beam to Euro code, lever arm (la) for K value of 0.12 = 0.81

Therefore As = $\frac{170x106}{0.87x410x0.81x467}$ = 1259mm^2

Provide 7Y16 (As = 1410mm^2)

Beam Section with Moment of 170kNm, Overall depth of 550mm, Width of 300mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ = $170x \frac{106}{25\ x\ 300x517x517}$ = 0.08<0.1569

Therefore, only tension reinforcement is required.

From Table 10.2, design of reinforced concrete beam to Euro code, lever arm (la) for K value of 0.12 = 0.9

Therefore As = $\frac{170x106}{0.87x410x0.9x517}$ = 1024mm^2

Provide 7Y16 (As = 1410mm^2)

Beam Section with Moment of 185kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ = $185x\frac{106}{25\,x\,250x467x467}$ = 0.14 <0.1569

Therefore, only tension reinforcement is required.

From Table 10.2, design of reinforced concrete beam to Euro code, lever arm (la) for K value of 0.14 = 0.81

Therefore As = $\frac{185x106}{0.87x410x0.81x467}$ = 1371mm^2

Provide 7Y16 (As = 1410mm^2)

Beam Section with Moment of 185kNm, Overall depth of 550mm, Width of 300mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ = $185x\frac{106}{25\,x\,300x517x517}$ = 0.09 <0.1569

Therefore, only tension reinforcement is required.

From Table 10.2, design of reinforced concrete beam to Euro code, lever arm (la) for K value of 0.14 = 0.89

Therefore As = $\frac{185x106}{0.87x410x0.89x517}$ = 1127mm^2

Provide 7Y16 (As = 1410mm^2)

4.6 Design of Doubly Reinforced Concrete Beam Using Code of Practice (Euro code 2: 1992)

Beam Section with Moment of 275kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ = $275x\,\frac{106}{25\;x\;250x467x467}$ = 0.2 >0.1569

Therefore tension and compression reinforcement is required.

Computing the ultimate moment of resistance of the concrete:

Mu = 0.1569bd^{2}Fcu = 0.1569x 250x467x467x25 = 214kNm

Effective depth from compression face of the beam (d^1) = 25 + 8+ 8 = 43mm

Overall effective depth = 467mm – 43mm = 424mm

Area of compression steel required = (275-214) x 10^6/0.95 x 410x 424 = 369mm^2

Provide 5Y12 (As = 566mm^2)

Euro code 2; 1992 clause 4.2.5 specifies a formula for computing the lever arm factor which is presented below:

Lever arm factor = 0.5 ± $\sqrt{0.25 - \frac{K}{0.9}}$

For doubly reinforced concrete beam, K value equals or exceeds 0.1569, for K equal to 0.1569, lever arm factor becomes 0.78.

Lever arm for the compression face of the beam becomes 0.78x effective depth = 0.78x467 = 365mm

Area of tension steel required = $\frac{214x106}{0.87x410x365}$ + 369 = 2012mm^2

Provide 7Y20 (As = 2200mm^2)

Beam Section with Moment of 275kNm, Overall depth of 550mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ = $275x\frac{106}{25\ x\ 250x517x517}$ = 0.16>0.1569

Therefore tension and compression reinforcement is required.

Computing the ultimate moment of resistance of the concrete:

Mu = $0.1569bd^2Fcu$ = 0.1569x 250x517x517x25 = 263kNm

Effective depth from compression face of the beam (d^1) = 25 + 8+ 8 = 43mm

Overall effective depth = 517mm – 43mm = 474mm

Area of compression steel required = (275-263) x 10^6/0.95 x 410x 474 = 65mm^2

Provide 2Y10 (As = 157mm^2)

Euro code 2; 1992 specifies a modification factor of 0.78 for computation of lever arm.

Lever arm = 0.78x effective depth = 0.78x517 = 403mm

Area of tension steel required = $\frac{263x106}{0.87x410x403}$ + 65= 1829mm^2

Provide 6Y20 (As = 1890mm^2)

Beam Section with Moment of 350kNm, Overall depth of 500mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 500-8-25 = 467mm

K = $\frac{M}{Fcubd2}$ = $350x\frac{106}{25\ x\ 250x467x467}$ = 0.26 >0.1569

Therefore tension and compression reinforcement is required.

Computing the ultimate moment of resistance of the concrete:

$Mu = 0.1569bd^2Fcu = 0.1569x\ 250x467x467x25 = 214kNm$

Effective depth from compression face of the beam (d^1) = 25 + 8+ 8 = 43mm

Overall effective depth = 467mm – 43mm = 424mm

Area of compression steel required = (350-214) x 10^6/0.95 x 410x 424 = 824mm^2

Provide 8Y12 (As = 905mm^2)

Euro code 2; 1992 clause 4.2.5 specifies a formula for computing the lever arm factor which is presented below:

Lever arm factor = $0.5 \pm \sqrt{0.25 - \frac{K}{0.9}}$

For doubly reinforced concrete beam, K value equals or exceeds 0.1569, for K equal to 0.1569, lever arm factor becomes 0.78.

Lever arm for the compression face of the beam becomes 0.78x effective depth = 0.78x467 = 364mm

Area of tension steel required = $\frac{214x106}{0.87x410x364}$ + 824= 2472mm^2

Provide 8Y20 (As = 2510mm^2)

Beam Section with Moment of 350kNm, Overall depth of 550mm, Width of 250mm and Moment Redistribution factor of 0.9.

Effective depth (d) = 550-8-25 = 517mm

K = $\frac{M}{Fcubd2}$ $= 350x \frac{106}{25\ x\ 250x517x517}$ = 0.21>0.1569

Computing the ultimate moment of resistance of the concrete:

$Mu = 0.1569bd^2Fcu = 0.1569x\ 250x517x517x25 = 263kNm$

Effective depth from compression face of the beam (d^1) = 25 + 8+ 8 = 43mm

Overall effective depth = 517mm – 43mm = 474mm

Area of compression steel required = (350-263) x 10^6/0.95 x 410x 474 = 471mm^2

Provide 5Y12 (As = 566mm^2)

Euro code 2; 1992 clause 4.2.5 specifies a formula for computing the lever arm factor which is presented below:

Lever arm factor = $0.5 \pm \sqrt{0.25 - \frac{K}{0.9}}$

For doubly reinforced concrete beam, K value equals or exceeds 0.1569, for K equal to 0.1569, lever arm factor becomes 0.78.

Lever arm for the compression face of the beam becomes 0.78x effective depth = 0.78x517 = 403mm

Area of tension steel required = $\frac{263x106}{0.87x410x403}$ + 471= 2301mm^2

Provide 8Y20 (As = 2510mm^2).

4.7 Check for Accuracy of Developed Design Charts and Tables

The check for accuracy of the developed design charts and tables was done by taking into cognizance certain deviation in design output obtained using the developed design tables and charts for both singly and doubly reinforced concrete beams. Deviation expressed as error in percentage was computed.

From the outcome obtained from accuracy check for the developed design charts and tables as presented in depicted in Table 4.4-4.7, it can be deduced that the developed design tables and charts for design of both singly and doubly reinforced concrete beams were reliable in their application for structural design of singly and doubly reinforced concrete beams as 50% of error calculated was found to be negligible (less than 1%). The discrepancy in value for area of steel required obtained using the developed design tables and charts and design code of practice according to Euro code 2: 1992 could be ascribed to subjective interpretation of the developed design tables and charts. Eventually, a reasonable agreement between the developed design tables, charts solution and Euro code 2: 1992 design solution was reached as depicted in Table 4.4 – 4.7 above.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The following deduction can be made from the findings obtained from formulation of design tables and charts for both singly and doubly reinforced concrete beams.

1 The design charts and tables were formulated using a mathematical derivation in accordance with design of reinforced concrete structures to Euro code 2.
2 The design charts and tables were formulated for a wide range of geometrical parameters expressed as non-dimensional parameters.
3 The use of non-dimensional parameters for formulating the design tables and charts narrowed down the design charts and tables to both singly and doubly reinforced concrete beams.
4 The developed design charts for structural design of doubly reinforced concrete beams account for reinforcement design of both tension and compression steel.
5 The developed design table for structural design of doubly reinforced concrete beams does not account for compression steel.
6 A reasonable agreement exists between the value for area of reinforcement required using the developed design tables and charts and Euro code 2 design solution.
7 The developed design charts and tables were adjudged to be efficient, effective and reliable for structural application.

5.2 Recommendation

Based on the outcome obtained from the formulation of design tables and charts for design of both singly and doubly reinforced concrete beams, the following recommendations can be made:

1 The formulated design tables and charts can be used for structural design of both singly and doubly reinforced concrete beams with varying cross section.
2 The developed design tables for structural design of doubly reinforced concrete beams cannot be applicable for reinforcement design of compression steel.

3 The developed design tables and charts will be valuable to undergraduate, post graduate, lecturers and practicing engineers at field and design office respectively.

4 Circumspect personal judgment must be applied in the interpretation of the formulated design charts and tables so as to minimize design flaws.

REFERENCES

Bayagoob, K. H., Yardim, Y., and Ramoda, S. A., (2013). Design Chart for Reinforced Concrete Rectangular Section. 2nd International Balkans Conference on Challenges of Civil Engineering, BCCCE, 23-25 May 2013, Epoka University, Tirana, Albania.

BS 8110 -1: (1985): Structural use of concrete: Code of practice for design and constructionBSI, 1985.

BS 8110 -3: (1985): Structural use of concrete: Code of practice for design and constructionBSI, 1985.

BS 8110 -1: (1997): Structural use of concrete: Code of practice for design and constructionBSI, 1997.

EN 1991-1-1:2003. Eurocode 1 : Actions on structures. Part 1-1: General actions – Densities, self-weight and imposed loads for buildings. CEN.

EN 1991-1-2:2003. Eurocode 1: Actions on structures. Part 1-2: General actions, actions onstructures exposed to fire. CEN.

EN 1992-1-1:2005. Eurocode 2: Design of concrete structures. Part 1-1: General rules and rules for Culverts. CEN.

EN 1992-1-2:2005. Eurocode 2: Design of concrete structures. Part 1-2: General rules - Structural fire design. CEN.

Reed, P., K. Schoonees, and J. Salmond 2008, Historic ConcreteStructures in New Zealand: Overview, Maintenance and Management,Science & Technical Publishing, Wellington, p. 90, (also seehttp://www.doc.govt.nz/upload/documents/science-and-technical/sap248.pdf, last accessed on 12 October 2012.

Reynolds, C. E., and Steedman, J. C., (2008). Reynolds's Reinforced Concrete Designer's Handbook. USA and Canada by Taylor & Francis 270 Madison Ave, New York, NY 10016, USA. Taylor & Francis is an imprint of the Taylor & Francis Group, an informa business.

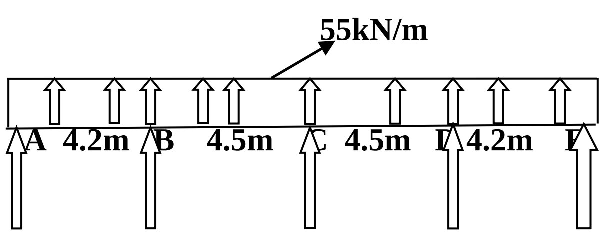
Shaeffer, R.E. 1992, Reinforced Concrete: Preliminary Design forArchitects and Builders, McGraw-Hill, p. 196.

Oyenuga, V. O., (2011). Simplified Reinforced Concrete Design. Surulere Lagos, Vason Concept Consultant.

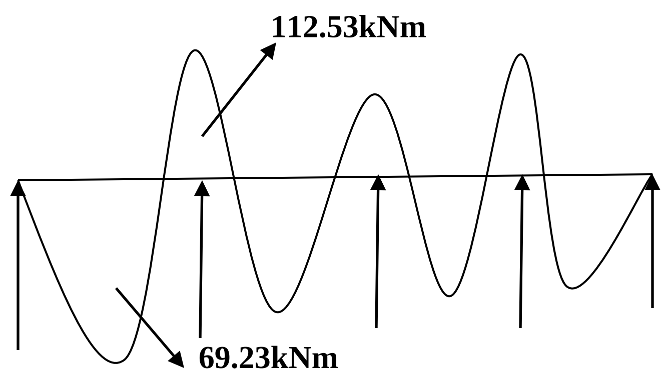
APPENDICES

APPENDIX A

Sample Design of Singly Reinforced Concrete Beam

Member Reference	Calculation	Output
	A four span continuous beam shown in the figure below support the following loads, itown self weight and finishes inclusive. Using Fcu = 25N/mm², Fy = 410N/mm², design the beam completely to Euro code. Dead load (Gk) = 24.6kN/m, Live load (Qk) = 14kN/m, Assume width of the beam to be 230mm and flange width of 780mm (Tee beam).	
Table 4.2, Euro code 2: 1992	**Ultimate design load = 1.35Gk + 1.5Qk = 1.35x24.6 + 1.5x14 = 55kN/m**	**55kN/m**
	 **d = 450 – 25-20 = 405mm** **Applying the ultimate load we have the loading as shown in the figure above and by symmetry MA = ME = 0.00** **Also by similar reasoning, MB = MD =?KNm, the value of this moment and Mc can be obtained**	

	using Clapeyrons three moments equation. **Applying Clapeyrons three moment equation we have:** **Span AB and BC** **$4.2MA + 2MB (4.2+ 4.5) + 4.5MC = 55/4 (4.2^3 + 4.5^3)$** **MA = ME = 0.00** **17.4MB + 4.5MC = 2354 (1)** **Similarly, 4.5MB + 18MC + 4.5MD= 2597 (2)** **4.5MC + 17.4MD = 2354 (3)** **Solving Simultaneously we have:** **MB = MD = 112.525kNm and MC = 88.015kNm** **Free Moments at mid spans:** **Span A-B and D-E = $0.125 (4.2^2)55$ = 125.685kNm** **Span B- C and C-D = $0.125 (4.5^2)55$ = 144.281kNm** **Using the principle of interpolation and assuming that the maximum moment occurs at the mid span the span moment are calculated as follows:** **Span A-B = $M^o - MA + \frac{MB}{2}$** **Span A-B = 125.685 – (0+112.525)/2 = 69.423kNm** **Span B-C = 144.281 – (112.525+ 88.015)/2 = 44.011kNm** **Span C-D = 144.281 – (112.525 + 88.015)/2 = 44.011kNm** **Computation of Shear Force** **VAB = 55x4.2/2 + 0-112.525/4.2 = 92.9kN**	
Table 10.2 Pp-86, Euro code 2: 1992	**VBA = 55X4.2/2 + 112.525-0/4.2 = 146.5kN** **VBC = 55x4.5/2 + 112.525-88.015/4.5 = 133.7kN** **VCB = 55X4.5/2 + 88.015-112.525/4.5 = 122.8kN**	

	VCB = VCD = 122.8kN **VDC = VBC = 133.7kN** **VDE = VBA = 146.5kN** **VBD = VAB = 929kN**  **Bending Moment Diagram** **Reinforcement Design** **Support B and D** **Moment = 112.525kNm** **K =** $112.\frac{525x106}{25x230x405x405}$ **= 0.119< 0.1569** **Therefore, only tension reinforcement is required.** **Table 10.2, Euro code 2: La = 0.84** **As =** $112.525\ x\frac{106}{0.87x410x0.84x405}$ **= 850mm^2** **Provide 3Y20 bar Top (As = 942mm^2)** **Span Reinforcement** **Here the flange will be in compression and all**	**Provide 3Y16mmBtm (As = 603mm^2)**

	that is needed is to check that the neutral axis is within the flange. This should be done by using the span with maximum moment. **Span A-B and D-E M= 92.949kNm** **b = 780mm, h = 450mm, d = 405mm** **K = 0.02 la = 0.95** **As =** $92.949\ x\ \frac{106}{0.95x410x0.84x405}$ **= 621mm**^{2} **Provide 2Y16mm + 1Y20mmBtm (As = 716mm**^{2}**)** **Span B-C** **Moment = 44.011kNm** **K = 0.14** **La = 0.95** **As = 294mm**^{2} **Provide 3Y16mm bars Btm (603mm**^{2}**)** **It is advisable to provide not less than 3Y16mm bars as main reinforcement in beams except in less structural members like lintel where 12mm bars can be used, however, for relatively short span, say less than 2.4m.** **Span C-D** **Moment = 78.742kNm** **K = 0.025** **La = 0.95** **As = 525mm**^{2} **Provide 3Y16mm bars Btm (As = 603mm**^{2}**)** **Serviceability Limit Check** **Deflection** **Basic Span/Effective depth ratio = 20**	**SProvide 2Y16+1Y20 Btm (As = 716mm**^{2}**)**

$\frac{100As}{bd} = \frac{100x716}{230x405}$ = **0.77%**

Modification factor = 1.3

Effective depth required = $\frac{4200}{1.3x20}$ **= 162<405mm**

Deflection is Okay.

Shear Design

Span A-B and D-E

V = 146.5kN

v = $146.\frac{5x103}{230x405}$ **= 1.573N/mm^2**

$\frac{100As}{bd} = \frac{100x942}{230x405}$ = **1.011%**

Vc = 0.63N/mm^2

Sv = $\frac{157(0.95)250}{230\ (1.573-0.63)}$ **= 172mm**

Provide 2legs Y10mm @ 150mmc/c

Span B-D

V = 133.7kN

v = $133.\frac{7x103}{230x405}$ **= 1.435N/mm^2**

$\frac{100As}{bd} = \frac{100x942}{230x405}$ = **1.011%**

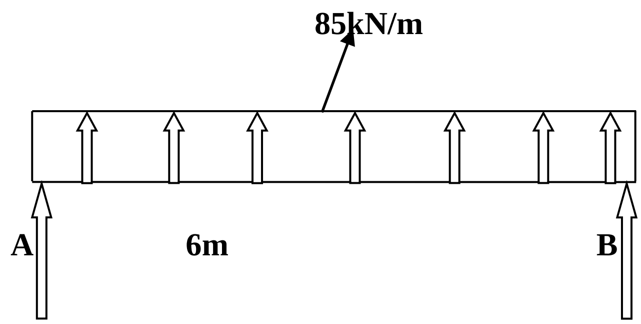
Vc = 0.63N/mm^2

Sv = $\frac{157(0.95)250}{230\ (1.435-0.63)}$ **= 201mm**

Provide 2legs Y10mm @ 200mmc/c

APPENDIX B

Sample Design of Doubly Reinforced Concrete Beam

Member Reference	Calculation	Output
	A simply supported beam carries an all inclusive ultimate load of 85kN/m and the beam is 6.0m long. Designing the beam to Euro code 2.  **Moment** **Try 450mmx230mm beam** **M = 0.125 (85)6^2 = 382.5kNm** **d = 450-25-20 = 405mm** **Mu = 0.156bd^2Fcu = 0.156x230x405^2x25 = 147.130kNm too low, hence increase the beam cross section to 600mm by 230mm and** **Mu = 0.156x230x550^2x25 = 271.343kNm** **K = $382.\frac{5x106}{25x230x550x550}$ = 0.220> 0.1569** **Therefore, tension and compression reinforcement is required.** **Steel Reinforcement Design in Compression** **$As^1 = \frac{(382.5-271.343)x106}{0.95(410)(550-50)}$ = 571mm^2** **Provide 2Y16mm + 1Y20mm Top (As = 716mm^2)** **Steel Reinforcement Design in Tension**	

	Lever arm for the compression face of the concrete = 0.78x550 = 427mm **As** = $\frac{271.343x106}{0.95(410)(427)}$ **+ 571mm^2 = 1632 + 571 = 2203mm^2** **Provide 5Y25mm bars Btm (As = 2455mm^2)** **Serviceability Limit State Check** **Deflection** **Basic Span/Effective depth ratio = 20** $\frac{100As}{bd} = \frac{100x2455}{230x550}$ **= 1.94%** **Modification factor = 1.6** **Effective depth required =** $\frac{6000}{1.6x20}$ **= 50<550mm** **Deflection is Okay.** **Shear Reinforcement Design** **V = 0.5(6)85 = 255kN** **v =** $\frac{255x103}{230x550}$ **= 2.016N/mm^2** $\frac{100As}{bd} = \frac{100x2455}{230x550}$ **= 1.94%** **Vc = 0.79N/mm^2** **Vc + 0.4= 1.19N/mm^2** **Using 2 legs of Y10mm bars we have:** **Sv =** $\frac{157(0.95)250}{230(2.016-0.79)}$ **= 132mm** **Provide 4-legs Y10mm bars @ 250mmc/c as links.**	